MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

Vol. 105 No. 4

MEETING OF 1945 JUNE 8

Professor H. H. Plaskett, President, in the Chair

The election by the Council of the following Fellows was duly confirmed:—

Frank J. Acfield, 61 Great Lime Road, Forest Hall, Northumberland;
Desmond Egan Andrew, A.R.Met.Soc., 29 Liverpool Road, Chester;
Roy Henry Garstang, Gonville and Caius College, Cambridge;
Alfred Llewelyn Middleton, B.A., 42B Fellows Road, Hampstead, London, N.W. 3;
Barrie Were Ashley Ricketson, Chestnut Cottage, Broadway, Worcestershire; and
Henry Wildey, 14 Savernake Road, Hampstead, London, N.W. 3.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Francis Holden, 1 Waterloo Terrace, Ashton, Preston, Lancashire (proposed by George J. Gibbs); and
Rev. Richard Calvert Wood, M.A., 55 Castle Street, Hertford (proposed by F. J. Hargreaves).

Forty-five presents were announced as having been received since the last ordinary meeting.

The President announced that the Council had elected the following as Associates of the Society:—

Professor William Otto Brunner, Director of the Eidgenössische Sternwarte, Zürich, Switzerland; and
Professor Marcel Gilles Jozef Minnaert, University, Utrecht, Holland.

MEETING OF 1945 JULY 13

Professor H. H. Plaskett, President, in the Chair

The election by the Council of the following Fellows was duly confirmed:—

Egil Kristoffar Bjørnerūd, Sagatūn, Hamar, Norway;
Peter Kerr, M.A., B.Sc., Woodland View, Hooton Road, Hooton, Cheshire;
Desmond Gerard O'Sullivan, B.Sc., 18 Virginia Gardens, Barkingside, Ilford, Essex;
Michael William Ovenden, 22 Coppetts Road, Muswell Hill, London, N. 10; and
Thomas William Wormell, M.A., Ph.D., Solar Physics Observatory, Cambridge.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

John Geoffrey Drewe Pratt, 4 Branstone Road, Kew Gardens, Richmond, Surrey (proposed by H. A. Brück); and
Frank Melson Wingate, Malcombe, Pine Wood Road, Bournemouth, Hampshire (proposed by J. W. Meares).

Forty-three presents were announced as having been received since the last ordinary meeting, including:—

A. J. Thompson, *Logarithmica Britannica* (formerly the property of Mr. Frank Robbins, presented by his daughter);
Album of photographs of Simeis Observatory after spoliation by the Germans (presented by Professor G. A. Shajn); and
Copies of various publications from Europe (provided by Dr. G. P. Kuiper).

The President referred to the signing by Council of the Diplomas for the two newly elected Associates and requested Dr. G. P. Kuiper, who was shortly to re-visit Holland, to convey the Diploma with the good wishes of the Society to Professor Minnaert. Dr. Kuiper expressed his willingness to comply with the request and gave some account of the hardships endured by scientists in Holland during the war.

The Meeting proceeded to a discussion on "The Origin of the Solar System", opened by Professor W. H. McCrea (on the theory of Dr. C. F. Weizsäcker) and continued by Dr. A. Hunter (on the theory of Professor H. Alfvén) and Mr. F. Hoyle.

ASTIGMATISM UNDER THE FOUCAULT TEST

E. H. Linfoot

(Received 1945 July 13)

Summary

The diffraction theory of the Foucault test developed in a previous paper is applied to discuss the appearances seen when the test is carried out on a circular mirror suffering from astigmatism of figure. The theory is shown to predict in a simple way the salient phenomena observed in practice, and an explicit solution in terms of the functions Si and Ci is obtained for the shadow-intensities corresponding to central and non-central settings of the knife-edge. This solution is used to estimate the limit of sensitiveness of the test as a means of detecting astigmatism.

1. *Introduction.*—In a previous paper * the diffraction theory of the Foucault knife-edge test was developed into a form which allows a theoretical discussion to be made of a number of questions of interest to the astronomical optician. Expressions were obtained for the intensities seen under the test on the disk of a circular mirror suffering from smooth errors of figure of arbitrary shape; they are valid except at points whose distance from the rim of the mirror is so small as to be comparable with the resolution of the viewing system (eye or camera) employed.† These expressions were applied to discuss the appearances under the test of a true mirror and of one suffering from zonal errors, and to estimate the sensitiveness of the test as a means of determining focal position and also as a means of detecting primary spherical aberration.

In the present paper, we apply the general theory to discuss the appearances seen when the test is carried out on a circular mirror suffering from astigmatism. A brief outline of the analysis leading to the fundamental formula (2.4) is given in section 2. In section 3 this formula is applied to the case of an astigmatic mirror and an explicit solution in terms of the functions Si and Ci is derived for the intensities seen under the test at the "preferred" focal setting which is used in practice. This solution is shown to predict in a simple way the salient phenomena observed in practice and is applied to estimate the limit of sensitiveness of the Foucault test as a means of detecting primary astigmatism.

2. *The Foucault Test.*—The arrangement of the test is represented in Fig. 1. We can cover all the cases where a convergent pencil is being null-tested by supposing that the wave-fronts originate at the surface of a nearly spherical mirror M which is being knife-edge tested at its approximate centre of curvature. The results of the test are then interpreted in terms of errors of figure of the mirror M. We suppose that these errors of figure may amount to several fringes, but that the error-slopes on the mirror, besides being free from discontinuities, are not so steep as to spread out the visible image to more than a moderate multiple, say 5 or 10, of the size of the Airy disk. Then it makes no appreciable difference if we suppose the wave to be leaving a true spherical

* *Proc. Roy. Soc.* In press.

† The intensity at points close to the rim depends critically on the particular characteristics (aperture and optical aberrations) of the viewing system and not predominantly on those of the mirror under test. Away from the rim the opposite is the case, and with the arrangements of the test usual in practice it is permissible to neglect the errors of the viewing system and the effect of its finite aperture.

surface M_0, lying everywhere within a few wave-lengths of the surface M, and suppose that at the point (x, y, z) on M_0 the complex displacement is

$$E(x, y) = | E(x, y) | e^{(2\pi i/\lambda)\phi(x, y)};$$

thus $| E(x, y) |$ is the amplitude at the point (x, y) on M_0 and $\phi(x, y)$ the phase there; the variation in $\phi(x, y)$ expresses the distortion of the wave-fronts. Outside the boundary circle C, lying in M_0 and corresponding to the edge of the mirror, we define $E(x, y)$ as zero.

We suppose that, after leaving the mirror, the light comes to a more or less imperfect focus in the neighbourhood of the centre of curvature O of M_0, at a distance s from its

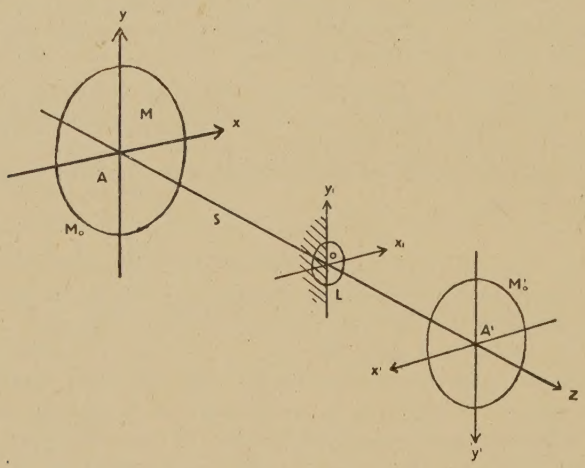

FIG. 1.

pole A. Through O, and perpendicular to AO, passes the plane of the knife-edge; x_1 and y_1, measured parallel to x and y respectively, are coordinates in this plane and in the space near it. For the present we suppose the knife-edge to occupy the part $x_1 \leqslant O$ of the plane; the effects of small variations in the position of the knife-edge will be considered later.

Just behind the plane of the knife-edge, a viewing system L receives the light and forms an image M_0' of the surface M_0; we denote by C' the boundary of M_0'. We define a system of coordinate-numbers (x', y') in the surface M_0' by assigning to the image P' of the point $P = (x, y, z)$ in M_0 the coordinate-numbers $x' = x$, $y' = y$. The intensity at P' is thus the same thing as the "intensity seen under the test" at P. To avoid unessential complications, we suppose that L is a perfect optical system.

By the usual arguments, based on an application of Huyghens' principle *, the complex displacement in the immediate image-surface, which we take to be a sphere, of centre A, touching the plane of the knife-edge at O is

$$W(u,v) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{ixu+iyv}E(x, y)dxdy \qquad (2.1)$$

where

$$u = \frac{2\pi x_1}{\lambda s}, \qquad v = \frac{2\pi y_1}{\lambda s} \qquad (2.2)$$

and the integral is only formally over an infinite domain, since the integrand vanishes

* For a fuller account of these arguments the reader is referred to the paper mentioned in the introduction.

outside the boundary-contour C. The complex displacement $D(x', y')$ at the point (x', y') in the final image-surface is obtained by integrating the product

$$\frac{1}{2\pi} e^{-i(x'u+y'v)} W(u, v)$$

over the part of the (u, v)-plane which is left uncovered by the knife-edge and falls within the aperture of the viewing system L. As was shown in the paper already referred to, it is permissible, except in the immediate neighbourhood of the boundary-curve C′, to neglect the effect of the finiteness of the aperture of L. The intensity at the point (x', y') in the final image-surface is then measured by the quantity

$$I = |\, 2\pi D(x', y') \,|^2, \tag{2.3}$$

where

$$2\pi D(x', y') = \int_0^\infty e^{-ix'u} du \int_{-\infty}^\infty e^{-iy'v} W(u, v) dv$$

$$= \pi E(x', y') + i \int_{-\infty}^\infty \frac{E(t, y')}{t - x'} dt \tag{2.4}$$

for all (x', y') not on C′, the last integral being interpreted as a Cauchy principal value when (x', y') is inside C′.

So far we have supposed the knife-edge to be centrally set in the plane through O. In the practical use of the Foucault test, however, an optician does not confine his observations to a single setting of the knife-edge. The information obtainable from the central setting is always supplemented by moving the knife-edge laterally to and fro and noting the changes in the appearance of the shadows. One reason for this practice is that an inspection of the shadows at a single knife-edge setting is not sufficient to give a correct idea of the run of the errors unless these are well below one fringe in amount all over the surface. When larger errors are present, the shadows are complicated by the presence of the dark fringes which have been investigated by Gascoigne *, and if observation is confined to a single knife-edge setting these fringes are likely to be misinterpreted as indicating local reversals of the error-slope. Again, the focal setting of the knife-edge is selected empirically in nearly all cases and is often readjusted during the work of figuring, a procedure which corresponds to varying by a very small amount the focal length aimed at in the finished system.

To calculate the effects of small displacements of the knife-edge on the observed intensities, we may express them as "induced errors" of figure on the mirror. Thus, to determine the effect of advancing the knife-edge across the z-axis through a distance c and increasing the focal distance by a small quantity δs, it is sufficient to replace $E(x, y)$ on the right of (2.4) by $e^{i\alpha(x^2+y^2)+ic'x} E(x, y)$ and $D(x', y')$ on its left by $e^{i\alpha(x'^2+y'^2)+ic'x'} D(x', y')$, where $\alpha = \pi\delta s/\lambda s^2$ and $c' = 2\pi c/\lambda s$. The resulting equation gives the new value of $D(x', y')$.

3. *Foucault Properties of an Astigmatic Circular Mirror; Sensitivity of the Test.*— We first establish the following properties, well known to experimental opticians, of an astigmatic circular mirror under the Foucault test:

(1) there is a unique focal setting at which the illumination seen on the mirror retains its left-to-right symmetry as the knife-edge is advanced across the image;

(2) the presence of astigmatism is inferred from any vertical asymmetry in the appearance of the mirror at this focal setting, which we shall call the preferred setting;

* *M.N.*, 104, 326–334, 1945. (2.4) above gives the extension to three dimensions of equation (2.8) of this paper.

(3) astigmatism shows itself most strongly when the mirror is oriented with its astigmatic principal directions at angles of 45° with the direction of the knife-edge. If one of these directions is parallel to the knife-edge, the appearance of the mirror under test is the same as that of a true mirror, and the astigmatism accordingly escapes detection.

To derive these properties from the general theory, we apply (2.4) to a mirror whose wave-function

$$\begin{aligned} E(x, y) &= e^{i(ax^2+2hxy+by^2)} \quad (x^2+y^2 \leqslant 1) \\ &= 0 \quad (x^2+y^2 > 1). \end{aligned} \tag{3.1}$$

If the plane of the knife-edge is at mean focus, the coefficients a, b satisfy the condition $a+b=0$, but as this is not the focal setting used in practice * we discard the condition and allow for variations in the knife-edge setting by writing

$$\begin{aligned} E(x,y) &= e^{i[(ax^2+2hxy+by^2)+\alpha(x^2+y^2)+c'x]} \quad (x^2+y^2 \leqslant 1) \\ &= 0 \quad (x^2+y^2 > 1) \end{aligned} \tag{3.2}$$

on the right of (2.4) and replacing $D(x', y')$ by $e^{i\alpha(x'^2+y'^2)+ic'x'}D(x', y')$ on the left. The parameter α then specifies the focal setting of the knife-edge, while $c'=2\pi c/\lambda s$ gives its lateral displacement c from the central position. Choosing $\alpha=-\frac{1}{2}(a+b)$ would correspond to setting the plane of the knife-edge at mean focus.

Applying (2.4) in this manner, we obtain for the complex displacement $D(x', y')$ the equation

$$2\pi D(x', y') = e^{i(ax'^2+2hx'y'+by'^2-c'x')}\cdot\left[\begin{matrix}\pi\\0\end{matrix}+i\int_{-\sqrt{1-y'^2}}^{\sqrt{1-y'^2}} e^{i[(a+\alpha)(t^2-x'^2)+2h(t-x')y'+c'(t-x')]}\frac{dt}{t-x'}\right] \quad (y'^2 \leqslant 1,\ x'^2+y'^2 \lesseqgtr 1);$$

$$= 0 \quad (y'^2 > 1). \tag{3.3}$$

When $\alpha=-a$, the square bracket is changed into its complex conjugate on writing $-x'$ for x' and hence

$$I(-x', y') = I(x', y');$$

when $\alpha \neq -a$ this relation no longer holds. Thus there is a unique focal setting at which the mirror shows a laterally symmetrical appearance for all values of c'. This preferred setting is the one used in practice and it is, broadly speaking, the focal setting at which the errors on the mirror are most plainly seen.

At the preferred focal setting we have, for $y'^2 \leqslant 1$, $x'^2+y'^2 < 1$ and for $y'^2 \leqslant 1$, $x'^2+y'^2 > 1$ respectively,

$$\begin{aligned} 2\pi D(x', y') &= e^{i(ax'^2+2hx'y'+by'2+c'x')}\left[\begin{matrix}\pi\\0\end{matrix}+i\int_{-\sqrt{1-y'^2}}^{\sqrt{1-y'^2}} e^{i(2hy'+c')(t-x')}\frac{dt}{t-x'}\right] \\ &= e^{i(ax'^2+2hx'y'+by'^2+c'x')}\left[\begin{matrix}\pi\\0\end{matrix}-i\int_{x'-\sqrt{1-y'^2}}^{x'+\sqrt{1-y'^2}} e^{-i(2hy'+c')u}\frac{du}{u}\right], \end{aligned} \tag{3.4}$$

which gives

$$\begin{aligned} I(x', y') = &\left[\begin{matrix}\pi\\0\end{matrix}-\mathrm{Si}\{(2hy'+c')(x'+\sqrt{1-y'^2})\}+\mathrm{Si}\{(2hy'+c')(x'-\sqrt{1-y'^2})\}\right]^2 \\ &+[\mathrm{Ci}\{|(2hy'+c')(x'+\sqrt{1-y'^2})|\}-\mathrm{Ci}\{|(2hy'+c')(x'-\sqrt{1-y'^2})|\}]^2, \end{aligned} \tag{3.5}$$

the second term on the right being understood, when $2hy'+c'=0$, to stand for its limiting value

$$\left(\log\left|\frac{x'+\sqrt{1-y'^2}}{x'-\sqrt{1-y'^2}}\right|\right)^2.$$

* Unless the mirror has its astigmatic axes at angles of 45° with the direction of the knife-edge.

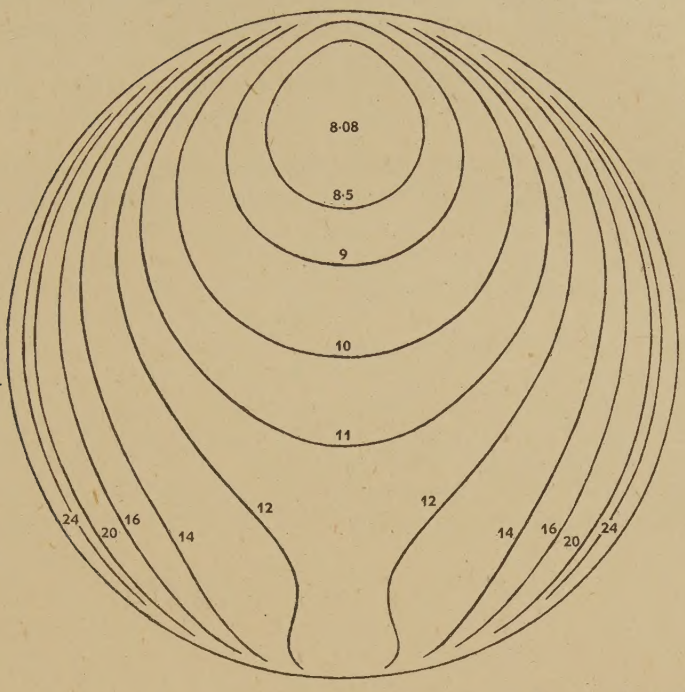

FIG. 2.—$\frac{1}{20}$ *fringe of astigmatism, tested with knife-edge centrally set. Isophotal lines.*

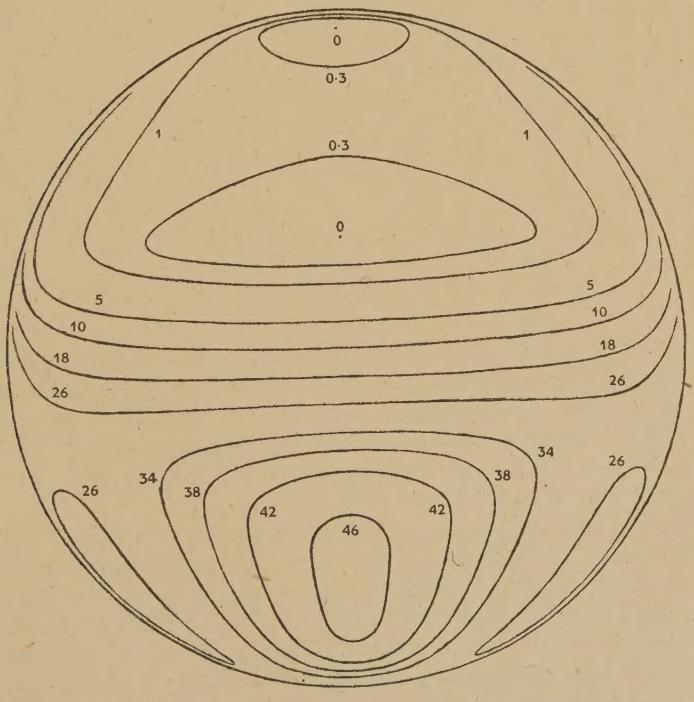

FIG. 3.—*One fringe of astigmatism, tested with knife-edge centrally set. Isophotal lines.*

Properties (1) and (2) have now been established. To prove property (3), we observe that of the coefficients a, h, b in (3.1) only h appears in (3.5). This means that a mirror for which

$$E(x, y) = e^{2\pi i h x y} \quad (x^2 + y^2 \leqslant 1)$$
$$= 0 \quad (x^2 + y^2 > 1) \tag{3.6}$$

and one of the figure (3.1) show the same intensities when each is tested at the preferred focal setting. The strength with which a given astigmatism shows itself under the test

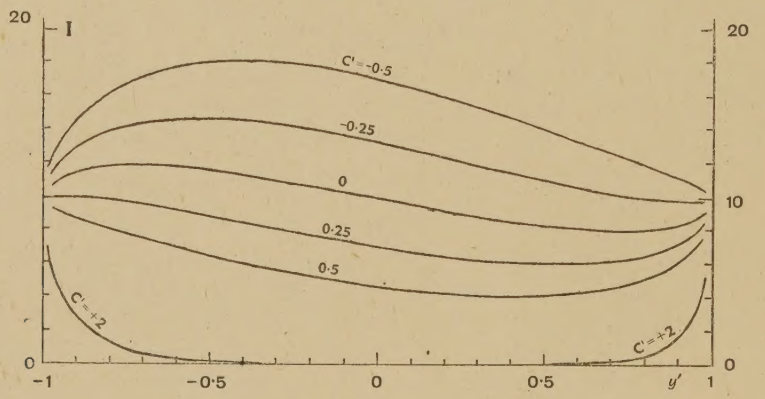

FIG. 4.

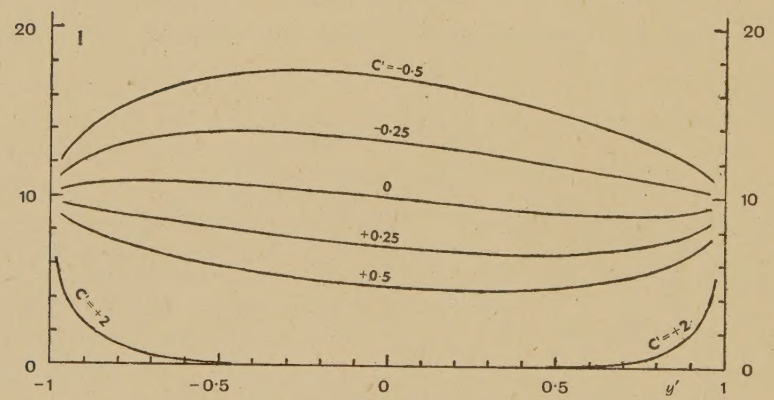

FIG. 5.

therefore corresponds to the size of the coefficient h in (3.1), and property (3) follows by a well-known result in coordinate geometry. For a mirror oriented so as to give h its maximum value, mean focus and preferred focus coincide, $E(x, y)$ is of the form (3.6) at the preferred focal setting, and $2h$ is 2π times the maximum optical path-difference at mean focus, measured in fringes. We suppose from now on that the mirror is so orientated and that $E(x, y)$ has this form.

As already remarked, the mirror darkens symmetrically from left to right, but not from top to bottom, as the knife-edge is advanced. Its appearance when the knife-edge cuts the axis ($c' = 0$) is shown for an astigmatism of approximately 1/20 fringe ($E(x, y) = e^{0 \cdot 3ixy}$) in Fig. 2 and for one of one fringe ($E(x, y) = e^{2\pi ixy}$) in Fig. 3. In the former case the vertical asymmetry is already very marked; in the latter the intensity-distribution is beginning to approach that predicted by ray theory, namely a dark upper semicircle and a fully lit lower semicircle. It is easy to verify from (3.5) that when the astigmatism

amounts to a large number of fringes the value of $I(x', y')$ is nearly π^2 below the line $y' = c/a$ and nearly zero above this line, while close to the line itself narrow dark fringes appear in the bright area. Thus equation (3.5) provides a simple means of predicting the observed phenomena, for large as well as for small amounts of astigmatism, when the test is carried out with a pinhole whose dimensions are small compared with those of the Airy disk.

To draw numerical conclusions about the limit of sensitiveness of the test, we must make some assumptions about the least departure from vertical symmetry in the intensity distribution at the preferred focal setting which can be detected by visual inspection. We shall assume, as in the previous paper, that a 40 per cent. difference in intensity between two fairly large, symmetrically disposed areas of the disk is easily detectable and that a 20 per cent. difference is detectable with certainty.

It is necessary to consider off-axis settings of the knife-edge, as well as the on-axis setting. On the other hand, it is clear from Figs. 2 and 3 that a knowledge of the intensity distribution down the vertical diameter $x' = 0$ will give a sufficiently good idea of its asymmetry for the purpose of the present estimations. Figs. 4 and 5 show this distribution, over a range of lateral settings of the knife-edge, for mirrors possessing 1/20 fringe and 1/40 fringe of astigmatism respectively. We conclude that the Foucault test, working at full efficiency, will detect 1/20 fringe of primary astigmatism with ease and 1/40 fringe with certainty.

Bristol:
The University,
1945 *July* 12.

NEW MEASURES OF THE SODIUM LINE D_1 IN THE SOLAR SPECTRUM

J. Evershed, F.R.S.

(Received 1945 September 17)

Summary

An extended series of measures of the sodium line D_1 in the solar spectrum compared with vacuum tube emission shows that sudden changes in the shift towards red of the solar line occur from time to time, at both centre and limb. These changes may be explained as due to varying movements of the sodium gas. The mean values of the shift Sun – vacuum tube show a small excess over the Einstein effect at the centre, and a larger excess at the limb, as occurs also in the *Fe* and *Ca* lines in the H and K region. The excess at the centre may be due to a general descent of sodium in the reversing layer, as is found in the higher levels of the solar atmosphere. But the mean excess at the limb is an outstanding discrepancy implying a larger shift of D_1 than relativity predicts.

The total shift of the line at the east and west limbs gives a value of the solar rotation at a high level in the reversing layer, and confirms previous results in showing an increase of angular speed with height above the photospheric level.

In a previous paper * I gave the results of measures of the sodium line D_1 compared with vacuum-tube emission, and I stated in the summary that "unlike the lines of iron or calcium, the sodium line is found to give the same shift towards red at the centre of the disk as at various points on the limb, and this shift agrees very closely with the Einstein relativity effect". These measures, made in the year 1937, were remarkably consistent, as is shown by the small probable errors resulting from a comparatively small number of spectra measured; and they agreed with previous measures made at Kodaikanal in showing no greater shift at the limb than at the centre of the Sun.

In a long series of subsequent measures of D_1 in spectra obtained with the multiple transmission liquid-prism spectrograph, it is now found that after a run of constant values a sudden change may occur in the shift or wave-length of the line, and this may occur in the spectra of the centre of the disk or in those obtained near the limb. Confining attention first to the centre of the disk, and to yearly values, I give the results shown in Table I.

From these figures it will be seen how greatly the shift varies from time to time. High values occur in May and June 1938, in May, June and the first half of July 1944, and February and March 1945. Low values closely follow the high in 1938 and 1944. In all three years the mean for the year is the same, and the general mean from all the spectra measured is +·0147 A. The relativity effect is +·0125 A., showing an excess of +·0022 A. I give in the fourth column the probable error of each set of measures in order to show roughly the consistency of each.

It is necessary to discover whether these variations are due to changes in the Sun or in the emission spectrum of the vaccum-tube. The D lines are sensitive to pressure, and according to measures by Humphreys at $3\frac{1}{2}$ to $10\frac{1}{2}$ atmospheres, the mean shift per atmosphere for D_1 is +·011 A. To confirm this I have compared the tube spectra assumed to be at a very low pressure with the open arc, and obtained a mean value, arc –

* *M.N.*, 98, 196, 1938.

TABLE I

Shift of D_1 at Centre of Sun

No. of Spectra	Date	Mean Shift A.	Probable Error
	Year 1938		
14	May 30 to June 10	+·0202	±·0006
8	June 14 and 24	+·0165	±·0008
11	July 3, 9 and 13	+·0090	±·0003
16	July 30, August 1, 3, 4, 5, 6	+·0121	±·0004
		+·0146	
	Year 1944		
29	April 30, May 3 to 28	+·0173	±·0018
10	June 19, 20, 22, 24	+·0152	±·0010
7	July 6, 13 and 14	+·0244	±·0016
11	July 15 to 20, and August 23	+·0061	±·0009
9	August 26 to 30	+·0123	±·0011
53	November 16 to December 29	+·0150	±·0004
		+·0147	
	Year 1945		
14	January 1 to 29	+·0122	±·0003
11	February 10 to March 12	+·0203	±·0001
31	March 13 to 30	+·0146	±·0005
23	April 3 to May 2	+·0139	±·0004
		+·0147	
	Mean of 252 spectra	+·0147 A.	
	Relativity effect	+·0125 A.	

tube, from 9 spectra of +·014 A. But it appears that the D lines are affected also by the density of the vapour, the co-called "pole effect". Thus when the density is great in the open arc and measures are made on the narrow self-reversals of the lines, the shift to red for both D_1 and D_2 is greater than when the emission lines are very narrow. In comparing the Sun with the vacuum-tube, there is also found to be a tendency to slightly larger values of the shift when the tube line D_1 is very narrow, and smaller values when it is wider than the solar line. I believe this is a density effect, and may have some slight influence in determining the amount of the shift of the solar line. It will not, however, account for the large changes seen in Table I, for on examination of the spectra giving the largest and the smallest shifts in the two years 1944 and 1945 there is found to be no relation between the shifts and the widths of the emission line.

There is the further possibility that rotation effects may come in if the slit of the spectrograph is slightly east or west of the centre of the disk, but any error in adjusting the Sun's image on the slit could scarcely equal one millimetre, and this amount of error in an image 60 mm. in diameter would give a shift of only ·0013 A., even in the most favourable conditions when the solar equator passes through the centre, whilst departures from the mean value may amount to as much as ·009 A. In other words, to account in this way for such a shift the slit would have to be at least 7 mm. from the centre, whereas it is certainly kept within 0·5 mm. during the short exposures.

These departures from the normal redward shift must therefore be due to changes in the Sun, and may readily be attributed to movements of the sodium vapour. The average excess over the relativity effect of ·0022 A. might be interpreted as a downward movement at this high level in the reversing layer, the sodium vapour descending on the average at about 100 metres per second; partaking therefore in the downward motion of the higher chromosphere represented by the lines H3 and K3 of calcium, in which the

TABLE II

Shift of D_1 near Limb of Sun

No. of Spectra	Date	ϕ	V_0 km./sec.	Mean Shift A.	Probable Error
	Year 1943				
29 pairs	Apr., May, June, July	2° to 30°	2·038	+·0175	±·0005
	Year 1944				
14 pairs	Apr. and Sept.	0° equator	2·034	+·0148	±·0010
32 pairs	Apr. to Sept. incl.	3° to 30°	2·028	+·0178	±·0004
22 pairs	Oct. to Dec. incl.	1° to 28°	2·015	+·0166	±·0005
19 single	Apr., Sept. and Dec.	90° north	...	+·0144	±·0010
23 single	Apr., Sept. and Dec.	90° south	...	+·0175	±·0006
68 pairs for rotation; 110 measures for shift			2·025	+·0166	
	Year 1945				
19 pairs	Jan., Feb., Mar., Apr.	0° to 30°	2·023	+·0180	±·0008
	Mean of 116 measures for V_0		2·028 km./sec.		
	Mean of 158 measures of shift		+·0169 A.		

excess shift over relativity implies a motion of 0·94 km./sec. descent.* This downward motion of sodium, however, is not constant, and may even be reversed for short periods.

Shifts at the Sun's Limb.—Coming now to measures near the Sun's limb, shown in Table II, these were undertaken to give values of the solar rotation at the level represented by the sodium lines, as well as the general shift towards red. The rotation is given by half the sum of the shifts at opposite ends of a diameter of the Sun's image, and the general shift to red by half the difference of the shifts, corrected by the equivalent in angstroms of the annual and diurnal motions of the Earth in the direction of the Sun, at the dates and times the spectra were photographed, as is of course necessary when any comparisons are made between solar and terrestrial wave-lengths. Every individual estimate of the shift depends therefore on measures of a pair of spectra, except in the case of the shifts at the Sun's poles, which were determined separately for each pole. For these, and for most of the spectra at the solar equator, a device for rotating the solar image through a measured angle was used, consisting of three plane mirrors placed some distance in front of the spectrograph slit.

The mean limb shifts for each of the three years 1943, 1944 and 1945, shown in Table II, are in good agreement, and the different series of measures during 1944 do not show any marked deviations from the mean value of all the measures. There are, nevertheless, large deviations within each series, especially those two which give the largest probable errors. These deviations might be accounted for as due to horizontal currents in the Sun. Errors of measurement by the positive on negative method are small, but perhaps they scarcely justify the inclusion of the fourth decimal in each series.

The mean shift of +·0169 differs from my previous results from 66 measures made in the year 1937, which was +·0123 A. I am unable to account for this difference: the 1937 plates are of first-class quality, with nothing to indicate that they might give low values, and a re-measurement of some of the plates confirms the previous results. In the case of the earlier Kodaikanal series of spectra, taken with the open arc, it is possible that a too small pressure correction had been applied to the shifts. According to my direct measures of the pressure effect of one atmosphere ·003 A. should be added, which would bring the result to +·018 A. for both centre and limb, but the Kodaikanal results are of low weight, owing to the small number of spectra measured and the relatively low dispersion of the grating plates.

It is probable that the mean value now obtained is itself an under-estimate, owing to

* *M.N.*, **91**, 267, 1931.

the slight tendency of the D emission lines to move towards red after the tube has been running for many minutes, and especially in an old tube that is nearly worn out. In the 1945 series I have used a new tube, run for short intervals, and this may account for the larger values shown in the Table. If this value +·018 A. is the true limb-shift of D_1, it implies an excess over relativity of ·0055 A., which compares with the excess shift of the iron and calcium lines in the H and K region, which I have found to be ·006 A. Any excess shift near the limb, or "limb effect", cannot be explained as a Doppler effect, since that would imply a recession of gas at opposite points on the limb, in other words an Earth effect. It therefore remains an outstanding discrepancy, implying a larger shift than general relativity predicts.

The Solar Rotation.—The values of V_0 shown in Table II are the results of measures obtained at the solar equator and at various latitudes between 0° and 30°, and these last have been reduced to the equator by means of a table constructed from the formula $(1{\cdot}50+0{\cdot}54\cos^2\phi)\cos\phi$ which represents the change of velocity with latitude with sufficient accuracy. The results are in good agreement in the three years, the mean for the total of 116 measures being 2·028 km./sec. This, compared with 1·94 km./sec. obtained at Kodaikanal in 1922–23 from iron lines in the violet region and 1·92 km./sec. from the Mt. Wilson measures between 1919 and 1924, indicates the increase in velocity with height above the photosphere, the sodium lines representing a higher level in the reversing layer than the iron lines. This increase applies also to the angular speed, for if the sodium line represents an increase of the solar radius of as much as 5000 km. compared with the level represented by the iron lines, the daily angular motion at the sodium level would be 14°·25 compared with 13°·73 for the iron region. Still larger angular speeds I have found with the hydrogen line α in the chromosphere, and with H and K in the prominences.*

Large variations occur in each series of measures, ranging between 1·9 and 2·2 km./sec. These are due to abnormal values at one or both limbs; and there is a tendency for a succession of high or low values to last for several days when the climate allows of a continuous daily record. It has been found in previous measures of rotation at different levels in the Sun's atmosphere that turbulent motion, causing variations in rotation, increases with height above the photosphere, culminating in the prominence region where the highest mean angular speed is found, and also the greatest irregular motion.

The limb spectra have all been photographed a short distance within the limb, usually at 0·98 from the centre. They have been secured under favourable atmospheric conditions, when the sky was free from thin diffusive clouds which have the effect of superposing light from the much brighter central part of the Sun's disk on the fainter light near the limb; but how much the rotation figures given in the table may still be affected by atmospheric scattering, even in a pure blue sky, could only be determined by measures obtained at high mountain stations, where for instance the corona may be photographed by Lyot's method.†

Highbroom, Ewhurst, Surrey:
1945 *September* 16.

* *M.N.*, **85**, 609, 1925, and **95**, 509, 1935.

† The effect of atmospheric scattering in altering the position of D_1 in the spectrum of the Sun's limb would appear to be quite negligible when the sky is clear, from the following experiment: a full exposure is made on the limb spectrum, but with the slit of the spectrograph crossing the limb and extending out into the sky beyond. When there is no diffusive cloud, no trace of the sky spectrum will appear on the photograph outside of the well-exposed limb spectrum. The superposition on the limb, therefore, of sky light which is too faint to make any impression on the plate, could not, it would seem, affect the position of the D line.

THE SOLAR ROTATION AND SHIFT TOWARDS RED MEASURED IN PROMINENCE SPECTRA

J. Evershed, F.R.S.

(Received 1945 September 26)

Summary

Measures of the shifts of the H and K lines in prominence spectra have been continued from April 1935 to March 1939. The results confirm previous measures in showing that the prominences give values of the angular speed of rotation in different zones of latitude greatly in excess of values derived from spectra of the reversing layer, or from the motions of sunspots. The mean value of the daily angular speed in this period was 16°·9, which exceeds the equatorial speed of the reversing layer by over 2 degrees.

The general shift of the H and K lines towards red in this series of measures only slightly exceeds the relativity shift of ·0081 A.

A comparison of the results with previous measures shows that the rotation values were about 2° per diem greater at times of maximum solar activity than near the minimum of 1933. The general shift has apparently decreased from +·015 to +·009 A. in the whole period 1926 to 1939, in which estimates have been made. The general mean of all the measures is +·012 A. or ·004 A. in excess of the relativity shift.

Continuing the series of measures of the H and K lines in prominence spectra, I have now to record the results obtained during the four years April 1935 to March 1939, covering the second period of maximum solar activity.

The auto-collimating spectrograph, consisting of solid glass prisms of 6-inch aperture, is the same as described in *Monthly Notices*.* The comparison spectrum is formed by a carbon arc containing traces of calcium, and giving the H and K lines of approximately the same width and intensity as the prominence lines. Only prominences showing narrow undistorted lines have been chosen for measurement, and since it very rarely happens that suitable prominences are situated at opposite ends of a solar diameter it is not possible to measure the double shift, east + west, as in the case of lines in the reversing layer. Each shift therefore of each individual prominence east or west is tabulated after correction for the Earth's motion. The tabulated shifts are then segregated in ten-degree zones of latitude, east and west separately. The half sum and the half difference of the total shift of each zone (W + E)/2 and (W − E)/2 give the mean rotation and mean shift respectively of each zone. The rotation shifts are then corrected for the retarding effect of the Earth's motion in its orbit, and for the inclination of the Sun's axis, and the shifts are corrected by adding ·0017 A., which is the mean pressure shift of H and K for one atmosphere. The results are set out in Table I.

The mean daily angle of rotation ξ in column 4 of Table I has been computed from the formula $\xi = \dfrac{SV \times 360^\circ}{2\pi R \cos\phi}$ (S = seconds in 24 hours, V = sidereal velocity in km./sec., R = Sun's radius, expressed in kilometres). In the computation the Sun's radius is

* *M.N.*, 95, 504, 1935.

TABLE I

Summary of Measures of Prominence Spectra, 1935–39

No. of Measures	Mean Latitude	V km./sec.	ξ Prominences	ξ Reversing Layer	Shift
	°		°	°	A.
29	10	2·525	17·7	14·4	+·0071
25	20	2·041	15·0	14·1	+·0086
63	30	1·826	14·6	13·7	+·0088
65	50	1·829	19·4	12·5	+·0091
Total 182			Mean 16·9		Mean +·0086

taken to be 960″ + 30″, the average height in each zone, excepting the 50° zone, in which the spectra were photographed at an average height of 40″. For comparison, column 5 in Table I gives the values of ξ from measures of lines in the reversing layer by Adams at Mt. Wilson in 1908, which agree very closely with the rotation values derived from sunspots. Later values, however, by St. John and others, give about half a degree less in each zone.

In all my estimates of rotation from prominence spectra there is no clear evidence of any decrease in ξ in the higher latitudes. I therefore give the mean value of all the zones, weighted according to the number of measures, at the foot of column 4 in Table I. The results confirm previous measures in showing that the angular speed in each zone of latitude always exceeds the speed of that zone in the reversing layer, and the mean of all the zones greatly exceeds the equatorial speed of the reversing layer, which according to Adams is 14°·5.

TABLE II

Period	No. of Measures	ξ	Mean Shift	Solar Activity
		°		
1926–28	381	18·1	·0153 A.	Maximum 1927–28
1929–34	401	15·5	·0104	Minimum 1933
1935–39	182	16·9	·0086	Maximum 1937–38

In Table II the mean values of rotation speed and line shifts are compared with previous values. From this it appears that at times of minimum solar activity the speed of rotation of prominences is least, and increases perhaps by two degrees at times of greatest activity.

The shift of the calcium lines H and K has apparently diminished from +·0153 A. in the first period to +·0086 A. in the last. This lowest value is close to the theoretical relativity shift, which at 30″ above the photosphere is +·0081 A. As it is impossible to say whether systematic errors due to the random motions of the prominences affect one series more than another, I can only give the general mean of the whole series of 964 spectra photographed in the years 1926 to 1939. This is +·0127 A., and this would be reduced to +·0119 A. if the less well-defined spectra in the earlier series were rejected. In round numbers, therefore, we may take the mean to be +·012 A., an excess over relativity or "limb effect" of ·004 A.

Highbroom, Ewhurst, Surrey:
1945 *September* 24.

NOTE ON THE HARVARD AND POTSDAM SYSTEMS OF SPECTRAL CLASSIFICATION

H. A. Brück

(Received 1945 July 4)

Summary

The spectral types of about 5600 stars in the zones $-15°$ and $-30°$ of the Potsdam Spektral Durchmusterung have been compared with their corresponding types on the Harvard system as given in the Draper and Yale Catalogues. The two regressions of the Harvard on the Potsdam system and *vice versa* have been derived for stars in two groups of magnitude and separately for the two Potsdam observers. The two classification systems are nearly the same for types near Ao, Ko and to some extent Go, whereas for types Fo or G5 on the Potsdam system systematic differences of about three-tenths of a spectral class are found. There is a magnitude effect, and there is also a minor systematic difference between the spectral estimates of the two Potsdam observers.

Comparisons of various systems of spectral classification have been published in recent years by Butler and Thackeray *, Vyssotsky †, and Seares and Joyner.‡ In Vyssotsky's paper there is found among other interesting comparisons a re-discussion of the data used by Mrs. Payne-Gaposchkin § for a comparison of Harvard and Potsdam spectral types in her study of the relation between spectral types and red-indices in southern selected areas. Earlier comparisons of the Harvard and Potsdam systems had been published in the course of statistical discussions of the distribution of spectral types by Fr. Becker ‖ and the present writer.¶

In view of certain discrepancies which have appeared, and in view of the importance of the comparison of different systems of spectral classification for any investigation of the relation between spectral types and colour indices, the correlation between the Harvard and Potsdam systems is to be discussed again in the present note on the basis of a more comprehensive material than has been used earlier; a separate comparison with Harvard is to be made of spectral types classified on the Potsdam system by Fr. Becker and the writer.

It may be recalled here that the Harvard and Potsdam criteria of spectral type as given in the Henry Draper Catalogue (HD) and in the Potsdam Spektral Durchmusterung of southern selected areas (PDM) ** differ in so far as in the Harvard classification "the general appearance of the spectrum" has been taken into account, whereas in Potsdam the assignment of spectral types is based on single numerical line ratios. In this case the lines hydrogen H_γ and H_δ, λ 4026 *He*, λ 4227 *Ca*, the K-line of Ca^+ and the G-band serve as standards whose relative intensities determine the spectral type. Intensity ratios of these lines can be arranged within a sequence which covers

* *M.N.*, **100**, 450, 1940.
† *Ap. J.*, **93**, 425, 1941.
‡ *Ap. J.*, **98**, 244, 1943.
§ *Ap. J.*, **90**, 321, 1939.
‖ *Sber. Ak. Wiss. Berl.*, **1932**, 86.
¶ *Z. Ap.*, **13**, 266, 1937.
** *Cf.* also *Trans. I.A.U.*, **V**, 181, 1935.

spectral types from early B to late K. Through comparisons of spectral estimates with the data of the Draper Catalogue this sequence has been fixed at the outset so as to provide maximum agreement between the two systems.

The following comparison of Harvard and Potsdam types is based chiefly upon stars in the 48 selected areas of the two zones $-15°$ and $-30°$ of the PDM which are common to both catalogues. In the zone $-15°$ Fr. Becker * has classified the spectra of altogether about 16,500 stars. The zone $-30°$ contains about 17,000 stars whose types have been determined by the writer.† The faintest stars classified in either zone are of magnitude $12^m.5$.

The spectral types of about 2600 stars in the zone $-15°$ and of about 2400 stars in the zone $-30°$ are found in HD. In addition to these there are available Harvard types determined by Miss Cannon for the Yale Catalogue ‡ which includes stars beyond the limit of magnitude of the HD. The Yale Catalogue contains additional Harvard spectra for some 300 stars of the Potsdam zone $-15°$ and for some 500 stars of the zone $-30°$. When stars whose types have not been estimated to the nearest decimal subclass, and others whose classification has been marked as doubtful are omitted, the spectra of altogether 5600 stars in the two zones are known on the Harvard and Potsdam systems and available for a statistical discussion. Nearly exactly half this number of stars belongs to each of the two zones which for the reason given have been discussed separately.

No distinction has been made in the comparison between giants and dwarfs which in the PDM in the case of bright stars of late type have been separated on the strength of the cyanogen absorption.

The correlation between the two systems of classification has been determined by the evaluation of the two regressions of Harvard on Potsdam spectral types and *vice versa*. These are given in the Tables I and II for stars of each subclass in the interval between the types B8 and K5. In order to test possible variations with magnitude the 2800 odd stars common to HD and PDM in each zone have been divided into two groups of stars brighter and fainter than $9^m.5$. The limit of magnitude in the second group is about $10^m.5$ on the Potsdam photographic scale of magnitudes. Two sets of regressions are therefore tabulated for each zone.

The Potsdam system proceeds everywhere in equal steps of a tenth of a spectral class, whereas the intervals between the subdivisions of the Harvard system are unequal. Stars earlier than B8 and later than K5 have been omitted from the Tables both on account of their small numbers and because at the two ends of the spectral sequence the comparison is too much affected by the asymmetrical frequency distribution of spectral types. In addition to the mean values of Harvard and Potsdam types $\bar{H}$ and $\bar{P}$ the Tables contain as measures of dispersion the mean square errors σ_H and σ_P expressed in units of a tenth of a spectral class, and the numbers of stars in each subclass. The unknown individual errors of the two classifications σ_1 and σ_2 are connected with the tabulated values of σ which measure the combined errors of the two systems by the relation $\sigma = (\sigma_1^2 + \sigma_2^2)^{1/2}$.

The average values of σ_H and σ_P which are given at the bottom of the corresponding columns of Tables I and II are sensibly the same. For stars brighter than $9^m.5$ they amount to about a quarter of a spectral class. They increase for fainter stars, and this increase is slightly more pronounced in the case of σ_H than in that of σ_P.

For some types σ_H and σ_P differ considerably from the average, as near Fo or G5 where high values of σ_H predominate. Since the individual classification errors of the two systems are different such fluctuations, which on account of the numbers of stars

* *Pub. Astr. Ob. Potsdam*, Nr. 92, 1938.

† *Ibidem*, Nr. 91, 1935.

‡ The parts of the Catalogue which have been used for the present comparison are contained in *Trans. Yale*, **11**, 1939; **12**, Part I, 1940; **13**, Part II, 1943.

TABLE I

Regressions of Harvard (H) on Potsdam (P) Spectral Types and vice versa for Stars Brighter and Fainter than $9^m.5$ in the PDM Zone $-15°$

	Stars $\leqq 9^m.5$			Stars $> 9^m.5$				Stars $\leqq 9^m.5$			Stars $> 9^m.5$		
P	$\bar{H}$	σ_H	Stars	$\bar{H}$	σ_H	Stars	H	$\bar{P}$	σ_P	Stars	$\bar{P}$	σ_P	Stars
B8	B8.1	± 2.2	16	B8.6	± 2.2	11	B8	B8.1	± 2.3	23	B8.3	± 2.4	36
9	9.0	1.5	45	8.3	1.4	18	9	9.2	0.9	48	A0.1	0.8	37
A0	A0.1	0.9	117	A0.4	1.5	195	A0	A0.0	0.9	101	0.5	1.6	157
1	1.3	1.2	24	0.3	1.7	35							
2	2.1	1.3	25	2.4	2.7	28	2	1.6	1.7	62	2.6	3.2	92
3	3.6	2.1	17	2.4	2.1	28	3	3.9	1.9	29	4.9	3.9	37
4	4.4	2.6	18	3.4	3.1	17							
5	4.9	2.8	33	6.4	4.8	40	5	5.0	2.2	40	5.4	3.9	38
6	7.9	3.7	8	7.7	5.6	10							
7	9.0	...	2	8.4	3.5	5							
8	9.2	3.5	24	F0.3	5.4	18							
9	9.4	2.3	5	3.0	...	3							
F0	F2.3	3.4	37	2.9	5.8	70	F0	8.6	2.2	48	8.1	4.6	39
1	3.3	2.5	16	5.1	6.0	19							
2	4.1	2.9	23	5.1	5.2	37	2	F2.0	3.6	33	F2.0	3.9	22
3	4.0	2.8	5	4.6	5.2	22							
4	5.9	2.1	7	6.3	2.6	11							
5	6.7	3.2	14	7.2	2.8	30	5	3.4	3.5	59	4.2	5.6	108
6	7.2	2.5	18	9.3	3.0	21							
7	8.0	3.0	14	7.4	3.2	21							
8	7.1	2.9	41	7.8	3.5	83	8	6.7	3.3	61	6.6	4.7	154
9	8.4	0.8	5	9.9	3.1	39							
G0	G0.6	3.0	21	G1.0	4.7	55	G0	G1.2	3.9	70	G1.0	4.7	220
1	1.1	4.2	9	1.0	3.2	26							
2	0.7	2.0	12	1.6	4.0	31							
3	0.4	2.0	7	1.9	5.8	40							
4	1.7	2.5	17	2.4	4.0	62							
5	3.6	2.2	11	6.4	5.1	53	5	6.6	3.5	83	6.5	4.1	253
6	5.0	2.7	7	6.5	5.0	41							
7	5.0	...	2	6.0	4.7	34							
8	7.5	3.3	59	8.2	4.1	157							
9	8.2	3.4	38	9.1	3.4	73							
K0	9.8	2.3	113	K0.2	3.2	257	K0	9.7	1.8	155	9.3	2.7	346
1	9.5	2.2	11	0.8	4.0	43							
2	K0.9	1.0	16	1.7	3.5	93	2	K2.2	2.2	57	K0.9	2.5	182
3	1.4	1.7	14	3.2	3.6	94							
4	4.1	2.4	25	3.8	3.9	60							
5	4.4	3.4	21	6.2	3.3	26	5	3.9	2.4	28	1.9	2.6	139
All stars		± 2.4	897		± 3.7	1906			± 2.6	897		± 3.4	1860

TABLE II

Regressions of Harvard (H) on Potsdam (P) Spectral Types and vice versa for Stars Brighter and Fainter than $9^m.5$ in the PDM Zone $-30°$

	Stars $\leqq 9^m.5$			Stars $> 9^m.5$				Stars $\leqq 9^m.5$			Stars $> 9^m.5$		
P	$\bar{H}$	σ_H	Stars	$\bar{H}$	σ_H	Stars	H	$\bar{P}$	σ_P	Stars	$\bar{P}$	σ_P	Stars
B8	B8·7 ± 0·7		16	A0·2 ± 1·5		6	B8	B9·3 ± 1·2		31	B9·7 ± 1·3		9
9	9·5	1·2	28	0·0	1·1	31	9	9·6	0·9	56	9·9	0·6	19
A0	A0·0	1·1	232	0·9	2·0	188	A0	A0·0	0·9	163	9·9	1·3	161
1	1·8	1·4	41	1·4	2·7	35							
2	3·1	2·4	28	3·1	3·2	23	2	1·3	1·6	92	A0·9	2·0	74
3	3·5	2·4	11	5·0	2·8	5	3	3·6	2·5	30	2·7	2·7	40
4	4·3	2·9	18	4·6	6·2	31							
5	4·1	2·4	21	7·3	4·5	32	5	4·1	2·4	27	4·4	2·6	40
6	7·4	3·8	9	8·2	3·9	21							
7	F0·0	...	5	F0·4	6·1	11							
8	A8·4	3·5	10	0·8	3·6	12							
9	F0·2	3·5	12	3·2	3·9	20							
F0	2·3	2·8	85	4·5	3·9	84	F0	8·4	2·3	58	7·4	3·6	57
1	5·3	2·2	14	5·5	3·1	30							
2	5·0	2·2	11	8·4	3·5	20	2	F0·6	1·9	42	8·1	3·8	37
3	4·4	1·3	5	6·7	2·6	13							
4	5·0	2·9	4	7·9	2·0	11							
5	4·9	3·4	10	8·1	2·2	10	5	3·8	3·7	80	F2·4	4·6	95
6	5·5	2·1	12	3·3	4·4	18							
7	6·5	1·9	23	8·4	2·8	47							
8	7·7	2·5	23	9·5	3·0	65	8	7·0	4·0	62	6·3	4·5	143
9	8·9	3·2	14	9·0	2·9	30							
G0	9·7	1·9	25	G1·4	3·9	43	G0	G3·2	4·6	58	G2·2	5·9	255
1	7·0	...	3	F9·7	2·7	15							
2	9·8	2·0	4	G2·3	4·3	24							
3	G0·1	2·8	10	1·9	3·8	16							
4	2·9	4·5	7	1·2	3·8	24							
5	1·0	...	3	2·6	3·4	19	5	8·4	3·0	84	7·5	4·2	199
6	2·0	2·4	5	2·8	3·5	19							
7	3·0	4·1	6	3·6	4·6	23							
8	3·1	3·1	13	5·1	4·6	44							
9	5·6	3·6	25	7·2	4·4	75							
K0	8·7	3·2	198	9·1	3·8	596	K0	K0·1	1·1	134	9·8	5·8	391
1	9·9	2·2	7	K0·1	4·5	49							
2	K1·3	1·0	12	1·9	3·7	63	2	1·5	1·9	52	K0·3	2·1	190
3	1·0	1·7	8	4·0	2·6	43							
4	1·9	1·4	8	4·5	3·1	24							
5	3·2	1·5	5	4·8	3·0	14	5	6·0	3·2	20	2·7	2·9	133
All stars		± 2·4	971		± 3·5	1834			± 2·4	989		± 3·2	1843

involved must be real, are not generally the same in σ_H and σ_P. No definite correlation has been found between the variations of σ and the inequalities of spectral intervals between adjacent subtypes such as Seares and Joyner have observed in their comparison of Harvard and Mount Wilson types.

Assuming the correlation between the two systems to be linear, the systematic differences between Harvard and Potsdam types can be estimated from the data given in

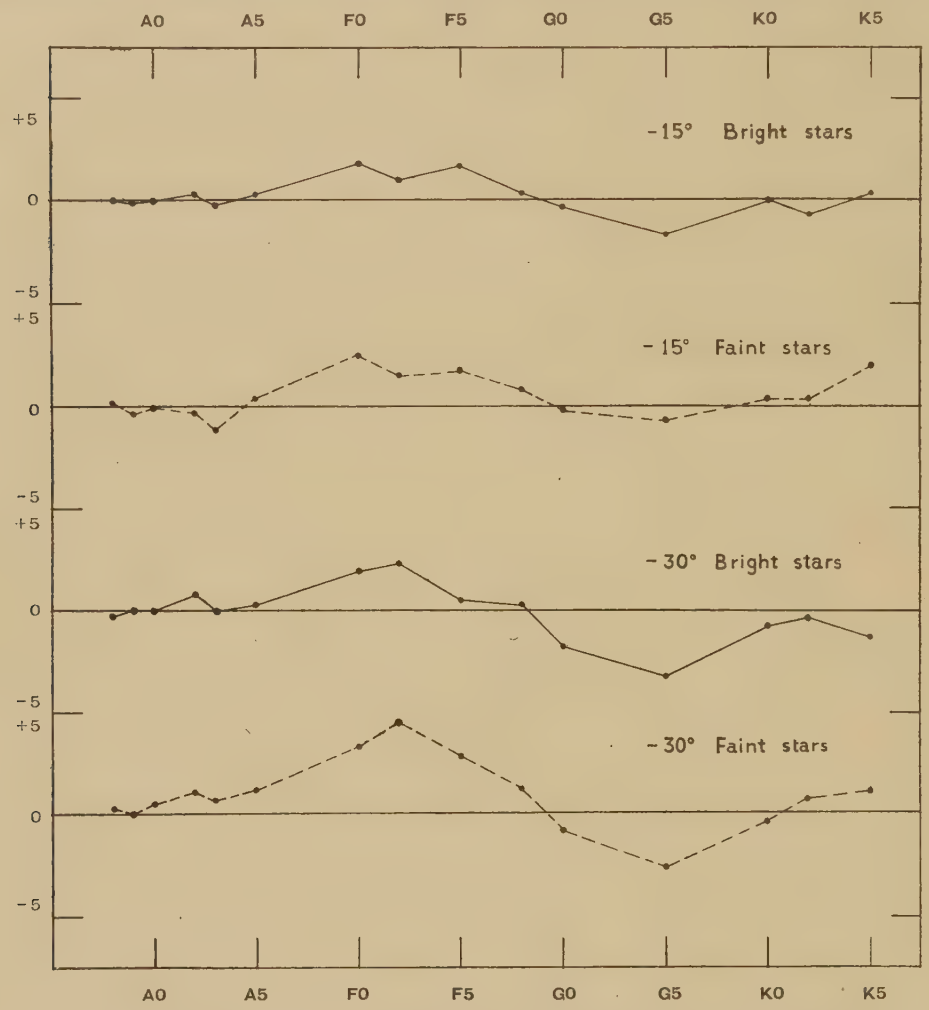

FIG. 1.—*Systematic differences between Harvard and Potsdam spectral types for the PDM zones −15° and −30° in which spectra have been classified by Fr. Becker and the writer. Ordinates give the values of (H − P) expressed in units of a tenth of a spectral class with Potsdam types as abscissæ.*

Tables I and II. Since in this as in any similar comparison of spectral classifications the value of the correlation coefficient is near to one, the actual differences between the two systems freed from statistical influences are readily obtained in the usual manner and with sufficient accuracy from the curves which lie midway between two corresponding regression lines.

The results are given in Fig. 1, where differences (H − P) are plotted against Potsdam types separately for the two Potsdam zones and for stars with magnitudes $\leqq 9^{m}.5$ and $> 9^{m}.5$, here called simply "bright" and "faint" stars.

The general trend of the curves is the same in all four cases, indicating that the Harvard and Potsdam systems agree for types near A0, K0 and to some extent near G0, whereas for types near F0 Harvard types are uniformly later, for types near G5 uniformly earlier than Potsdam types. These differences are greater for stars in the $-30°$ zone where spectra have been classified by the writer than in Fr. Becker's $-15°$ zone.

The curves also reveal a minor magnitude effect which cannot be explained by changes in the regressions with magnitude on account of changes in the frequency distributions of spectral types in so far as for types in the spectral interval between A5 and G5 and also for K5 the differences (H – P) become more positive with decreasing brightness. For some types such as F2 and G5 on the Potsdam system there exists a systematic difference between the spectral estimates of the two Potsdam observers which may amount to two-tenths of a spectral class.

These results disagree to some extent from values of corrections which Mrs. Payne-Gaposchkin (*loc. cit.*, Table 5) has determined and adopted to convert Potsdam into Harvard types. There is, however, general agreement between the spectral differences of Fig. 1 and the results of Vyssotsky's statistical rediscussion of part of Mrs. Payne-Gaposchkin's material in which he has studied the spectra of about 1300 stars with magnitudes between $10^m.0$ and $10^m.9$ in the PDM zone $-45°$ where spectra have been classified by Fr. Becker. The present results for zone $-30°$ agree surprisingly well with a preliminary comparison of the two systems by the writer for which only about 700 stars were used in seven fields which had been selected at random among the 24 fields of the zone. The spectral differences of Fig. 1 which are based on the reduction of a considerably greater material of spectra than previous comparisons should be used whenever Potsdam types of stars brighter than about $10^m.5$ photographic and classified by any of the two Potsdam observers are to be converted into Harvard types.

Observatory,
Cambridge:
1945 July 1.

THE FABRY METHOD OF STELLAR PHOTOMETRY

R. O. Redman

(Received 1945 September 15)

Summary

A discussion is given of experience gained at the Radcliffe Observatory during the past five or six years in the use of the Fabry method of stellar photometry. An accurate light curve of the eclipsing variable RS Sgtr has been determined by this means. The average deviation from the mean light curve for one exposure was $0^{m}.013$. Experiments have been made to observe by continuous recording, instead of by discrete exposures, but little improvement of accuracy has so far been achieved, except on nights with a poor sky. Results obtained in checking a magnitude sequence in Harvard Standard Region E3 are given, and compared with four sets of independently determined magnitudes. Of the five sets of measures, those made by the Fabry method show the smallest average deviation from the mean.

The photometric method first proposed by Fabry for measuring the brightness of the sky *, and later suggested by him also as a means of stellar photometry, appears to be generally less well known than it deserves, especially since in some circumstances it may offer decided advantages over other procedures. It has been used at the Radcliffe Observatory during the past five or six years for a number of problems in stellar photometry and the experience gained is recorded here in the hope that it may be of assistance to others. In South Africa the method has also been used successfully by Cousins †, while in addition it is at present being employed at the Cape Observatory. Grouiller ‡ has given a useful account of the method fairly recently, although there is comparatively little overlap with the present work. And since this paper was prepared in substantially its present form, news of another publication by Fabry has been received §, although up till now it has been available to the present writer only in abstract.‖

Optical Considerations.—The optical procedure is simple: by means of a small auxiliary lens, with a focal length of the order of one inch, which we shall call the *Fabry lens*, placed in or near the focal plane of the telescope, the telescope objective is photographed in the light of a selected object, which may be a star, a cluster, a nebula, or even empty sky. Apart from temporary fluctuations due to scintillation, and some secondary effects to be mentioned later, this arrangement gives an image, the *Fabry image*, of uniform surface brightness, very suitable for accurate photometry. If the telescope already has that rare virtue, the power to give really good extra-focal images, there is little point in using the Fabry method. In most cases, however, the Fabry image has a flatter light distribution than the normal extra-focal image, while the method also is capable of succeeding where even a perfect telescope would in practice fail, e.g. in giving images of uniform light distribution from irregular sources such as a star cluster or nebula.

Generally speaking the method is applied most easily to telescopes of fairly large *f*/number. The choice of the Fabry lens is usually dictated by the required speed,

* *Ap. J.*, **31**, 394, 1910. † *M.N.*, **103**, 154, 1943. ‡ *Annales d'Astrophysique*, **3–4**, 90, 1939.
§ *Annales d'Astrophysique*, **6**, 65, 1943. ‖ C.D.A.L., *Bulletin* **20**, 1945.

which depends not only on the amount of light available, i.e. on the object observed and the telescope aperture, but also on the area of plate illuminated. This is determined by the relative aperture of the telescope and the focal length of the Fabry lens. Other things being equal, the accuracy increases with the size of the image, which should thus be kept as big as possible. In practice, however, it will be found that there is not much to be gained by using an image diameter greater than about 1 mm. At the Radcliffe Observatory two telescopes have so far been used: (1) a 7-inch refractor working at $f/17$ (actually one of the finders of the 74-inch telescope), and (2) a 12-inch reflector working at $f/18$, temporarily installed on the 74-inch mounting. The Fabry lenses have varied in focal length between 5 and 14 mm. With the 7-inch telescope, having an old objective whose transmission is low by modern standards, the 14-mm. lens gives a Fabry image

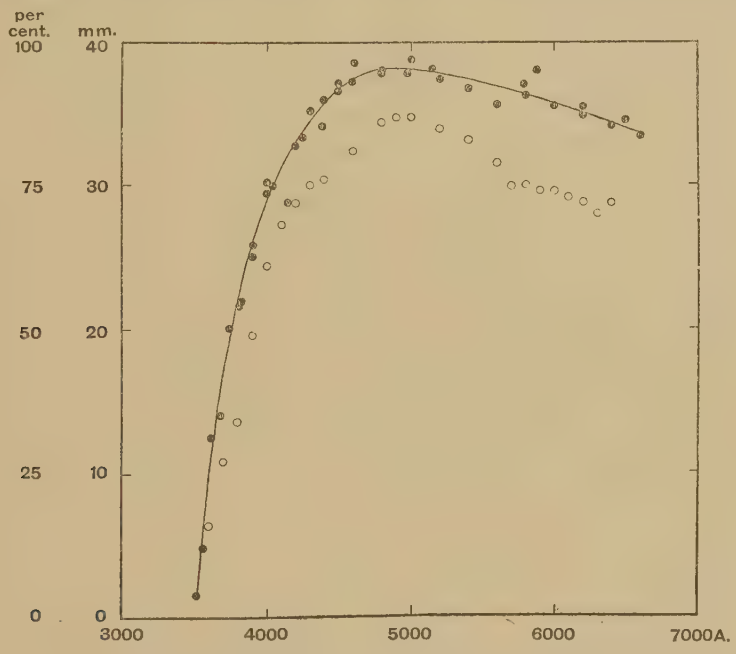

Fig. 1.—*Colour curve (filled circles) and transmission at centre (unfilled circles) of 7-inch objective. The colour curve is convex to the objective.*

of diameter 0·83 mm., and the exposure time for a 6th magnitude B-type star on an Ilford Zenith plate, without a filter, is 1½ minutes.

The aperture of the Fabry lens has to be sufficient to include the whole of the telescope image of the star or other object whose brightness is to be measured. Where a refractor is being used to observe individual stars, the important quantity is the chromatic aberration of the telescope objective. The necessity for care here may be emphasized by illustration. The 7-inch telescope, a visual refractor, has a rather steep colour curve (Fig. 1) so that at the best visual focus light of wave-length λ 3600 forms an extra-focal patch more than 1½ mm. in diameter. For this wave-length the measured transmission at the centre of the objective is about 0·16, so that there is an appreciable amount of such light present, especially when a blue star is being measured, and it makes some contribution to the Fabry image, although naturally quite invisible to the eye. Thus, if the Fabry lens is placed near the visual focus, it needs a clear aperture of at least 2 mm. for photographs taken without a filter, although the observer may be strongly tempted to use a much smaller diaphragm than this, in order to reduce the amount of light from the sky, or to exclude neighbouring stars.

For stellar work the Fabry lens need not be corrected; a simple lens will generally be found to give perfectly satisfactory results. Indeed, such a lens has advantages of its own, for it is essential to keep the glass perfectly clean, and it is important to avoid small defects such as bubbles in the glass. Furthermore, a wide selection of simple lenses is immediately at hand in the observatory's stock of eyepieces. Any lens, however, should be examined with a magnifier before being put into use, and none should be accepted with visible defects in the glass, or in the surface polish. Such faults are not uncommon in old eyepieces, although much rarer in those of modern manufacture.

An uncorrected lens is usually sufficient, because the focal length is small, the aperture is small, and because it is not called upon to form an image containing any very fine detail. Owing to plate grain and the fact that the extreme edge of the image should in any case be avoided during measurement, a circle of confusion of diameter about 0·04 mm. can be tolerated. Now consider a simple plano-convex lens of crown glass, with focal length 14 mm. and aperture 2 mm. The refractive index at λ 4900 may be about 1·523, at λ 3600 about 1·539. The resulting difference of focal length is about 0·42 mm. If a mean focus is used, the maximum longitudinal chromatic aberration is 0·21 mm., with a corresponding diameter for the circle of confusion 0·03 mm., which can be neglected. It is unlikely that a wave-length range giving a variation of refractive index much greater than this will be used at once, so that in most practical cases the Fabry lens need not be achromatic. For an object at infinity, which is a sufficiently near approximation to the actual conditions, longitudinal spherical aberration with this lens would be, according to Danjon and Couder *, about 1·08 h^2/f, where h is the radius of the lens aperture, and f the focal length. With $h=1$ and $f=14$, this becomes 0·08 mm., only about one-fifth of the chromatic aberration. Of the other defects of a simple lens, only one need be brought into consideration, namely the field distortion which causes the magnification to vary with distance from the optical axis ("pin-cushion" or "barrel" distortion, according to the position of the diaphragm). In the case of the 14-mm. lens this was measured by photographing a grid placed over the telescope objective and was found to be entirely negligible.

A simple lens, such as is recommended for general use, may be less satisfactory in special cases. For instance, if it were desired to measure the integrated brightness of a large globular cluster or nebula, the aperture would need to be relatively great, while the focal length would need to be kept fairly short, on account of the low average surface brightness. Such cases are rather unusual and are best considered individually. Let us inquire briefly into the extreme example of the large globular cluster 47 Tucn, whose diameter, according to Shapley †, is 54′, the present value of the integrated brightness being $4^m{\cdot}5$. To measure the magnitude of this cluster, the Fabry lens would need to cover a circle of sky about 1° in diameter, and over this area the average surface brightness of the cluster plus sky is little greater than of empty sky by itself. If exposure times are not to exceed say one hour, the Fabry lens would probably have to work at $f/2$ or $f/3$. With the 7-inch telescope, a lens of aperture at least 50 mm., or 60 mm. to be quite safe, would be required. Suppose a simple plano-convex lens of crown glass were used, of aperture 60 mm. and focal length 150 mm., giving a Fabry image of diameter about 8·8 mm. Chromatic aberration, estimated as before, would give a circle of confusion with maximum diameter about 1·8 mm. and spherical aberration about 2·6 mm., so that the image would have a very fuzzy edge. However, it would be of such large area that only a small fraction need be measured, and possibly even in this extreme example, by measuring only over a diameter about 4 mm. at the (Fabry) image centre, satisfactory results could be obtained with an uncorrected lens.

Although the advantages of using a simple Fabry lens have been stressed, this does not mean to suggest that properly corrected lenses, if available and of good quality,

* *Lunettes et Télescopes*, 139, 1935.
† *Proc. Nat. Acad. Sci.*, **30**, 63, 1944.

should not be used. Nevertheless it needs to be realized that much good work can be done with the simplest possible equipment.

It will be clear that in general only one object can be observed at a time, for even if there are several stars in the field covered by the Fabry lens, there will only be one image on the plate. This, an advantage when dealing with the integrated brightness of an extended object, is an outstanding disadvantage when dealing with individual stars, and makes the Fabry method unsuitable for mass production work. It is true that something can be done by the aid of a number of Fabry lenses used simultaneously in different parts of the telescope field, as has been tried by Lagrula *, and also at the Radcliffe Observatory, but this solution saves observing time only at the expense of a good deal of practical complication.

Distribution of Light in the Fabry Image.—The definition of the Fabry image in any normal case depends wholly on the Fabry lens, and not on the optical performance of the telescope. This forms probably the greatest advantage of the method. The equivalent of practically perfect extra-focal images can be obtained from a telescope of very indifferent optical performance. A visual refractor may be used for photographic work, and good images can also be obtained off the telescope axis. An important case is the use of a reflector in its Herschelian form, i.e. with no secondary flat and with the Fabry apparatus just far enough off the axis of the parabolic mirror to clear the incoming light beam. For stellar work the aperture of the Fabry lens is then settled by the amount of coma and astigmatism in the image, chromatic aberration being absent. The method is practicable only for mirrors of fairly large *f*/number.

Temporary disturbances in the distribution of light in the Fabry image are caused by scintillation, but these are so transient that unless exposures are much shorter than would be required in any ordinary observational programme, they will be entirely smoothed out in the photography. With a very bright star, very brief exposures, say of the order of 0·01 second, would show "shadow bands" of precisely the same nature as those seen at solar eclipses. Normally, however, the Fabry method is independent of scintillation, an advantage when compared with photometric methods using in-focus images, as has been pointed out by Cousins.

Although the definition in the Fabry image may be good, the distribution of light across the image may yet not be perfectly uniform. To examine this question the distribution was measured with a microphotometer across three different diameters of eleven images on a plate of RS Sgtr and HD 166197, dated 1940 April 16, and not taken specially for this purpose. The microphotometer slit length was restricted to about one-fifth of the diameter of the image. Plate grain was not smoothed, and most of the scatter in the measures (Fig. 2) is due to this. Although under ordinary visual inspection the images appear sharp and uniform, there is in fact a fair amount of rounding off near the edge. There are a number of reasons for this. Aberrations of the Fabry lens, diffraction, scattering in the photographic emulsion, and the finite resolving power of the microphotometer all make small contributions (the last named being of little importance). The possibility of excessive diffraction, due to focusing the star too sharply on the Fabry lens, and hence using this at what amounts to a very large *f*/number, is in practice ruled out by the aberrations of the objective and by the normal imperfections of seeing. The edges of these particular images have probably suffered a little from the use of a 1-mm. diaphragm, which must be too small, although at the time of taking the plates this was not realized, the colour curve of the objective being measured only some time later. There must have been a certain small amount of ultra-violet light, forming a relatively large extra-focal patch on the Fabry lens, and subject to a certain amount of vignetting by the diaphragm. Later observations avoided this difficulty, by using either a larger aperture or a filter to cut out the ultra-violet.

Another factor which may affect the light distribution in the Fabry image is varying

* Grouiller, *loc cit.*, 91.

transmission of the telescope objective from centre to edge. In the case of the 7-inch telescope this has been examined (*a*) by direct measurement at λ 4100, at the centre of the objective and at a point three-quarters of an inch from the edge, (*b*) by photographing the objective with an ordinary camera, f.l. 13·5 cm., in place of the Fabry lens, using the zenith sky as source and an ultra-violet filter, giving with this particular set-up a mean wave-length about λ 3900. Since the transmission of the objective had earlier been measured throughout the entire spectrum, it is possible to convert the results from (*b*) into equivalent distributions for other wave-lengths, assuming that the centre-edge

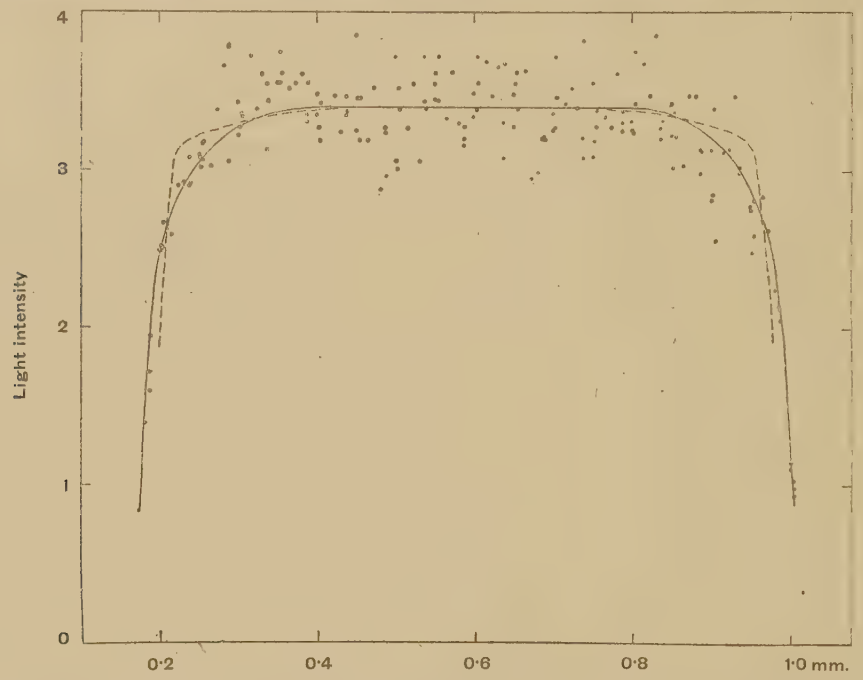

FIG. 2.—*Measured distribution of light across a Fabry image (plate grain unsmoothed). The dotted curve gives approximately the distribution which would result from variable transmission of the telescope objective alone.*

variation arises solely from varying thickness of glass. At λ 4100 the converted figures agree fairly well with those from (*a*). The measured distribution itself suffers from some rounding at the edges, but this is comparatively small and probably for the greater part is due to errors of measurement. The transmission curve has an inverted saucer shape and there is no convincing evidence that it has contributed materially to the rounded edges of the Fabry image. The effect of this variable transmission has been indicated in Fig. 2 for λ 4400, which has been computed by Knox-Shaw as the "effective wave-length" for a B-type star with this instrument, with Ilford Zenith plates and no filter. Were a large refractor used, especially if work were being attempted in the violet or ultra-violet, it is possible that the centre-edge variation of transmission of the objective would become of greater importance.

If the density distribution in the image is not perfectly flat, systematic errors may be introduced into the measures. If D is photographic density and I is light intensity,

$$I=f(D),$$

but, in general, if $\bar{I}$ is the average light intensity in a non-uniform image, and $\bar{D}$ is the corresponding average photographic density arrived at in the measures,

$$\bar{I}\neq f(\bar{D}).$$

The size of the error depends on the part of the plate characteristic curve which is being used, and the range of D over which an average is being taken. $f(D)$ is in practice not represented by a simple algebraic expression, so that it is virtually impossible to lay down general criteria for the attainment of a given accuracy. It is essential to avoid measuring right to the edge of the image, and a good rule is to keep always in mind the possibility of errors entering at this point, if necessary conducting appropriate tests from time to time, to ensure that serious trouble is being avoided. It has been verified that in the conditions used for the measurement of the plates of RS Sgtr and HD 166197, one of which provided the data of Fig. 2, no systematic error greater than $0^{m}.01$ has been introduced from this particular source.

Minor Instrumental Details.—Before passing on to questions of procedure, a number of comparatively small points may be mentioned.

(1) Should a filter be used it will probably be placed between the telescope objective and the Fabry lens. If more than a quite small distance from the latter, a fairly sharp image of the filter will be thrown on the plate. Under these circumstances it is essential not only to use a good quality filter, but also to keep it scrupulously clean. On the other hand it is not desirable to place the filter too close to the Fabry aperture, in case of small local irregularities (although modern filters by reputable makers can be astonishingly uniform). A distance of the order of 5 cm. from the Fabry lens is suitable.

(2) Guiding need not be so accurate as for ordinary in-focus photography, especially if comparatively bright stars are being observed, allowing the use of a fairly big diaphragm. With fainter stars, however, it is desirable to use as small a diaphragm as possible, as otherwise the amount of light from the sky becomes comparable with that from the star itself, and then the star image has to be centred and held on the aperture with a certain amount of care. If a fairly good telescope mounting is being used, little guiding may be necessary during the exposures themselves, but since changes from one star to another may be frequent, it is desirable to have some arrangement by which the image can be centred on the Fabry lens without disturbing the plate. In the case of the Radcliffe 7-inch, a second telescope of $4\frac{1}{2}$ inches aperture, the other finder of the 74-inch reflector, has been used as guider. Cousins, who used a similar method for his work, has pointed out that in this case multiple cross-wires in the guider are a great convenience. With the 12-inch mirror there was no quick method for setting, while guiding was done on other stars in the field. Grouiller has described a guiding arrangement involving a diagonal unsilvered mirror.

(3) There should be some convenient means of changing the diaphragm on the Fabry lens, since it is sometimes necessary, as will appear below, to measure the sky brightness, and this needs a bigger aperture than is generally desirable for a star.

(4) The most fruitful source of bad images is wrong positioning of the Fabry lens with respect to the telescope focus, or poor centring of the telescope image on the Fabry lens aperture.

(5) It will be clear that the plate holder should be provided with cross motions, permitting a large number of exposures to be made on one plate, preferably in a rectangular array near the plate centre.

Photometric Procedure.—The earliest application of the Fabry method at the Radcliffe Observatory was made, at the suggestion of the present writer, by the late E. G. Williams to the problem of the colours of the B-type stars of the southern sky. A description of his work has already been published.* Williams's procedure was not exactly the same as that used in the present work, where plates have been calibrated with a tube sensitometer. It is unnecessary to go into details concerning the construction of this well-known apparatus. We need only remark that the "tubes", in our case holes drilled in a solid block of wood, should be at least ten times as long as the diameter of the largest aperture, and that they should be made so that as little light as possible is reflected

* Williams and Knox-Shaw, *M.N.*, **102**, 226, 1942.

from their walls on to the plate. A convenient method is to thread each tube internally, which at once cuts down the reflected light and aids in the insertion and adjustment of whatever diaphragms are considered necessary. Precautions must be taken to ensure that the illumination of the sensitometer apertures is uniform. A further point is that it is desirable, in order to minimize photographic errors such as the Eberhard effect, to make the sensitometer spots the same size as the Fabry images.

While a tube sensitometer, used with proper care, is capable of giving results with a very high degree of internal consistency, it may introduce small systematic errors in one way or another. The most likely source is the fact that the wave-length of light used in the sensitometer is not exactly the same as that of the light reaching the plate in the telescope, although the difference can, and should be, minimized by a proper choice of filters. The trouble varies with the type of plate and the wave-length region involved; it is most likely to be noticeable with colour sensitive emulsions. In variable star work such an error is unlikely to become serious, but in setting up a magnitude sequence there might be a cumulative error, perhaps of the order of one per cent., exceeding the accidental error and yet not detectable except by checking with some independent method. Cousins has expressed the opinion that a sensitometer should not be used for establishing the scale of a magnitude system, and it is undoubtedly true that a procedure by which the calibration is done at the telescope, using the light of the stars themselves, is to be preferred. For use with the Fabry method either a step wedge of sputtered platinum or rotating sectors may be suggested. The sector method is at present under trial at the Cape Observatory by Stoy and Davies.

Of other factors occasionally cited as giving rise to small errors in photographic photometry, telescope exposures and sensitometer exposures have generally been kept equal in length, possible changes of the latent image between exposure and development have been rendered unimportant by making the interval between star and calibration exposures short in comparison with the interval between these and development, while variations of humidity and temperature between telescope and calibrating apparatus have generally been fairly small.

All plates have been brushed during development. A strong M.Q. developer was used. The formula, one employed for many years at the Cambridge Observatory, contains a rather small amount of potassium bromide, 0·7 gm./litre. Subsequent tests have shown it to be improved by increasing this to 3·5, or even 5 gm./litre. (Compare Ilford ID-2 with 2, or Kodak D19 with 5 gm./litre.)

A standard microphotometer may be used for measuring the plates, but such elaborate apparatus is not strictly necessary. It is important, however, that slightly less than the full area of the Fabry images should be measured, and that only this area should be illuminated by the photometer. At the Radcliffe Observatory a projection type measuring machine * required only minor modifications to give very satisfactory service in this task. In its original form the illumination in this machine was by means of a projection lamp, an image of whose filament was thrown on the plate by a condenser. By interposing near the plate a short focus simple lens, an image of the condenser was instead projected on the plate, and by the insertion of suitable diaphragms over the condenser the area of plate illuminated could be varied at will. For locating particular images a more general illumination could be simply attained by inserting a small piece of ground glass immediately behind the plate. The screen on which the plate image is normally projected was replaced by one with a number of interchangeable apertures, behind which is a photocell, connected to a conventional one-valve amplifier and galvanometer.

Correction for Sky Light.—Any light which passes directly through both the telescope objective and the Fabry lens contributes to the measured image. This will include a certain amount of light from the area of sky projected on the aperture of the Fabry lens, together with a small amount of light from other sources, scattered into the main optical

* *M.N.*, 99, 686, 1939.

path by dust on the telescope objective, or by other imperfections. These contributions are added to the light of the star or whatever object is under observation. The importance of this false light varies a great deal, according to particular circumstances, but in general its amount must be determined and, if necessary, a correction applied to the measures. In the case of a bright star, or where we are merely establishing equality between two similar stars, the correction is not needed; if we are measuring stars over a wide range of brightness, including some fairly faint objects, or if we are determining the integrated brightness of an object of low average surface luminosity, such as a nebula, the sky correction cannot be omitted. What constitutes a "bright" or "faint" star from this point of view depends on the telescope used, on the aperture of the Fabry lens, and on whether the Moon is in the sky. Measures made on moonless nights with the 7-inch telescope, with a Fabry lens of focal length 6 mm. and aperture 1·5 mm. (about 1·7 minutes of arc), gave a mean value for the sky fog of $11^m.80$. This was with Ilford Zenith plates and no filter, and is the mean of five nights, individual values varying between $11^m.39$ and $12^m.26$. Some of this variation is probably real. The measures were made by photographing first a star of known brightness, about 11^m, and then empty sky. The figure given is equivalent to $12^m.0$ per square minute of arc, and is fairly certainly higher than the true intrinsic brightness of the night sky, for it includes faint stars not individually discernible in the telescope (the exposures were taken not far from the Milky Way) and also a certain amount of false light, of which this instrument shows rather a great amount.

Clearly it is an advantage to reduce the sky light by using as small an aperture as possible for the Fabry lens. With this particular telescope an improvement has been attained by the use of an æsculine filter, which cuts out all the ultra-violet. This has permitted the diaphragm to be reduced to about 0·4 mm. (0·5 minute of arc) for work in the ordinary photographic region with individual stars. In most cases the larger the telescope aperture, the less important the sky fog becomes, although this does not hold with large extended nebulæ or clusters. In the matter of the correction for sky light, as indeed in other respects as well, there is a close similarity between the Fabry method and ordinary photo-electric photometry, the photocell aperture and the Fabry lens aperture being equivalent to each other.

The Light Curve of RS Sagittarii.—As an example of the results which can be obtained with the Fabry method, a light curve of the eclipsing variable RS Sgtr has been determined. Only one comparison star has been used, namely HD 166197, which is about a tenth of a magnitude fainter than the variable at maximum light, and is of the same spectral type, B5. Observations were made with the 7-inch telescope, using the Fabry lens of 14 mm. focal length, at an aperture 1 mm., with Ilford Zenith plates and no filter. The effective wave-length has been estimated to be 4400 A. 1162 exposures of $1\frac{1}{2}$ minutes each were made, usually in groups of four, each group preceded and followed by four similar exposures on the comparison star. 18 exposures were rejected after measurement.

Corrections for atmospheric extinction were applied, determining the extinction coefficient by means of the apparent variation of the comparison star with zenith distance. Although sufficient for the present purpose, the correction being small, it was later found that as a means of determining the extinction this method is very inaccurate. The reason is that in Pretoria during calm weather, particularly in the dry season, there is almost always a systematic, and sometimes very considerable, increase in transparency during the early hours of the night. Smoke and dust begin to settle at sunset, as atmospheric turbulence dies down, and the effect is accentuated by the fact that a dominant feature of the South African atmosphere, especially during winter, is an inversion of temperature associated with the descent of air brought in by high-level winds from equatorial regions. Were it not for ventilation by this clean air, the transparency of the sky in the interior of South Africa during the dry season would probably be much inferior to what it really is.

A correction for sky fog in this particular case was necessary only on bright moonlit nights. It was determined by opening the Fabry aperture to 5 mm. and giving an exposure on empty sky, also of 1½ minutes. The ratio of light transmitted by the two apertures was determined both geometrically and photometrically. The two values were in rather poor agreement with each other, which is thought to be due to the fact that the larger aperture was limited not by a diaphragm, but by the lens mount, which is more difficult to measure, since in other cases, where two or more diaphragms have been compared, better agreement has been reached. Hence the photometric determination has been preferred. However, because the variable never differs by more than one magnitude from the comparison star, while both are fairly bright, the final correction is always small and the maximum difference between the figure based on the geometric ratio and that based on the photometric ratio is only $0^m.01$.

Fig. 3 gives the observed light curve, each point there, with a few exceptions, representing four exposures. The mean deviation from a smooth curve, not shown in the figure,

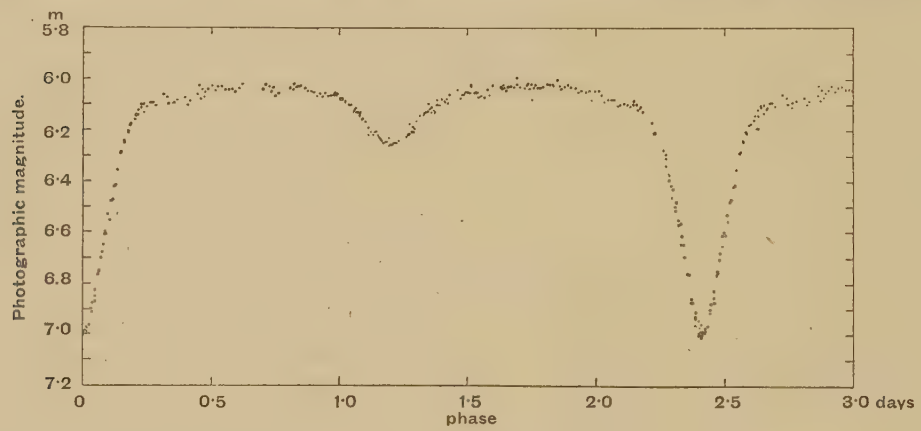

Fig. 3.—*Light curve of RS Sagittarii. With a few exceptions each plotted point represents four exposures.*

is $0^m.013$ per exposure. As far as these observations can decide, the orbit is circular. Further details are tabulated below. It is hoped to publish a photometric orbit in a separate paper.

		h m	° ′		
RS Sgtr	HD 167647	18 11·0	−34 08 (1900)	B5	
Comparison	166197	04·4	33 49	B5	$6^m.6$ pg.

Mean epoch of observations	JD 2429542.
Assumed period	2·41568 days.
Depth of primary minimum	$0^m.97$.
Depth of secondary minimum	$0^m.22$.
Brightness at maximum light	$6^m.0$ pg.

Continuous Recording.—The light curve just given for RS Sgtr appeared so satisfactory that the question came to mind whether an improvement could be realized by making a continuous record of the light of variable and comparison stars, instead of relying on discrete exposures. One advantage would be that the stars would be observed simultaneously instead of alternately, avoiding the necessity of frequently moving the telescope, and thus saving an appreciable amount of time. Another would be that well-marked changes of sky transparency could be detected more easily and either avoided or some correction might possibly be made. Also photographic errors should be reduced, as local fluctuations in the record could be smoothed over. The method should show to

greatest advantage in studying variables of short period, or where the presence of rapid light changes, not necessarily regular, is expected.

Some trials were made on 29 Canis Majoris with the 12-inch Burch mirror (which is aluminized). It should be said at once that this is not a very suitable star for the purpose, since the period is rather long, 4·4 days, and there is a poor selection of comparison stars available. Furthermore, the spectrum is peculiar, with variable emission lines, which complicate the interpretation of any ordinary light curve, while it is possible that the light variation is not perfectly regular.

An experimental apparatus was built with smooth guides, on which a large plateholder could be driven at constant speed by means of a fairly accurate screw and a synchronous clock motor. The guides were steel rods ground smooth with oil and carborundum, using a split collar as tool. The mounting of the carriage on the rods was geometrical. In the base three Fabry lenses were mounted, each of focal length about 14 mm., all three with coarse adjustments, and two with double eccentric fine adjustments, permitting them to be centred respectively on the variable and the two comparison stars, HD 59612 and 60532. The whole apparatus was held on the 74-inch telescope mounting, just far enough off the axis of the 12-inch mirror to avoid the incoming light beams. The mirror was diaphragmed to a square, one edge of which was parallel to the direction of plate motion.

The use of plates was dictated by war-time conditions, but it seems likely that cinematograph film, if available, would be preferable, allowing better and neater design of the apparatus. An instrument of this kind needs to be provided with a rather wide range of speeds, to control the exposure, which may be achieved most conveniently by some form of friction drive. However, this was not done on the experimental set-up used here, and the exposure had to be adjusted, either by cutting down the square mirror aperture to a rectangle, not a very desirable method, as it makes the demands for accuracy of plate drive more stringent, or by choosing a filter which happened to make the exposure of about the right strength. The combination of Ilford Zenith plates and a green filter was found suitable here, the light used being then restricted to a rather narrow region in the blue-green. The plates were calibrated by making a contact print of an optical wedge direct on to the emulsion, the wedge image being of about the same width as the trail of the Fabry image. The trails were measured in the microphotometer in the normal way.

The accuracy of the results was rather disappointing. The chief source of inconsistency was apparently the comparison star HD 59612. There is no good reason to suspect the star itself; most of the difficulty is believed to have come from the corresponding Fabry lens, which was taken from a rather old eyepiece and has a definitely poor surface polish, so that small movements of the star image across the lens aperture are likely to be followed by changes in the amount of light transmitted. If this star is rejected, and only HD 60532 used as comparison, the internal consistency of the measures is improved considerably, and, furthermore, they then agree better with some earlier work using discrete exposures. On the other hand, they show no noticeable superiority to the latter, except on nights with passing clouds, or a patchy sky, which would normally be avoided for accurate photometry. This failure to surpass the discrete exposure method may be an indication that the present limit of accuracy is in the photographic plate, rather than in the sky, although even in this case one would expect some improvement to follow the introduction of continuous recording. But the discrete exposures have a rather important advantage; they are always taken through the same lens, and small variations of focus due to lack of flatness of the plate, or lack of parallelism between plate and cross-slide on which the plateholder is carried, or other small imperfections in the apparatus, are largely nullified if variable and comparison stars are being continually alternated. This compensation, however, does not work when each star has its own separate Fabry lens. Means for getting over such trouble could be devised. The use

of film running over accurately machined rollers on good bearings should be an improvement, while it might be possible for an artificial Fabry image from a lamp to be photographed alongside each stellar Fabry image.

The continuous recording method, if observations are carried on in runs of an hour or two, is much easier for the observer, who is spared the task of frequently moving the telescope from one star to another, and of starting, stopping and timing innumerable exposures. It also offers decided advantages when rapidly varying stars are being observed in poor sky conditions. Yet in general, for normal variable star work, the additional complication involved hardly seems worth while, unless a considerable improvement in accuracy over the ordinary Fabry method can be achieved.

Checking Standard Magnitude Sequences.—Another problem in which the Fabry method has been found of use is in the matter of standard magnitudes for the southern sky. A rather extensive photometric programme is at present being undertaken by the Cape and Radcliffe Observatories in co-operation, and a part of the work consists of setting up convenient and accurate standard sequences, with magnitudes on the international photographic or photovisual scales. It is not necessary to discuss here the general problem of stellar magnitudes, so delightfully simple in theory and so miserably complicated in practice, fundamentally important in all astrophysics and yet so neglected by twentieth-century astrophysicists. More can be said on the subject when work still in progress is ready for publication. It is sufficient meanwhile to say that magnitude determinations in the northern sky can usually be made by direct comparison with the North Polar Sequence, which in practice provides the definition of the stellar magnitude system. The problem for southern observers is to set up in the southern sky a sequence, or sequences, of comparable accuracy and on the same magnitude system.

It may be assumed that over the range of magnitudes with which we are at present concerned, roughly $7^{m}.0$ to $11^{m}.5$ photographic, the international magnitude scale *for stars of identical colour* is an accurate Pogson scale, i.e.

$$\text{magnitude} = -2.5 \log_{10}(\text{apparent brightness}) + \text{constant}.$$

The constant defines the zero of the scale, which is conventional, and which at present we ignore. We concentrate first on testing certain sequences, to see that their scale satisfies this relation accurately, which is a necessary, but not sufficient, condition that the magnitudes should be on the international scale.

As a part of the work, groups of stars of the same spectral type (F for preference) have been chosen and their relative magnitudes determined by the Fabry method. The programme is still in progress, so that the magnitudes given below, of a group of A-type stars in E3, should not be regarded as necessarily the final word on the subject. They were determined with the 7-inch telescope, using a Fabry lens of 5 mm. focus, at aperture 0·4 mm., with Ilford Zenith plates and an æsculine filter. The stars were taken in small groups, each covering a range $1^{m}.0$ to $1^{m}.5$ and the sequence built up by the aid of numerous overlaps. The construction of a magnitude sequence by small steps such as this is open to some criticism, as a small curvature of scale is difficult to avoid. The procedure may be likened to testing an optical flat with a proof plane of much smaller diameter. It may, however, be safely employed if checked by other methods, including at least one which bridges a larger magnitude interval in one step. The criticism is not essentially one of the Fabry method itself.

A sky correction was applied, assuming the sky light to be $12^{m}.0$ per square minute of arc (see above), all observations being made on moonless nights. The maximum value of the correction was $0^{m}.03$. Accidental errors are higher than with RS Sgtr, on account of the smaller images. The material does not allow an accurate determination of the mean error, but if comparing stars *b*, *c*, *d*, etc., with *a* by single exposures in the order *a b a c a d a*, etc., and assuming *a*'s magnitude, the average error per star is about

$0^m.025$ under good conditions. This error is really a measure of the internal consistency of the magnitude difference $x - a$, derived from one exposure of x and two of a.

The accompanying Table I gives the results. The figures in brackets are the number of Fabry exposures for each star. There are four other groups of magnitudes given for

TABLE I

Photographic Magnitudes in E3

Cape Zone Catalogues	Type	Fabry	*Pg'*	Sector	Præsepe	*H.A.* 89	Mean
3763	Ao	7·44 (4)	7·48	7·42	7·59	7·46	7·48
3746	Ao	7·60 (4)	7·60	7·59	7·67	7·59	7·61
3738	B9	7·92 (3)	7·92	7·93	7·89	7·87	7·91
3676	Ao	8·10 (4)	8·13	8·18	8·11	8·00	8·12
3740	B9	8·19 (2)	8·15	8·20	8·14	8·03	8·16
3758	Ao	8·28 (4)	8·31	8·34	8·26	8·34	8·31
3768	Ao	8·35 (7)	8·33	8·38	8·30		8·34
3726	Ao	8·55 (6)	8·61	8·55	8·45		8·54
3727	Ao	8·80 (4)		8·82	8·78		8·80
3809	A3	8·85 (2)	8·92	8·86	8·86	8·86	8·87
3678	A2	9·60 (1)		9·67	9·57		9·61
3725	A3	9·65 (1)		9·66	9·60		9·64
3703	Ao	9·70 (3)		9·68	9·67		9·68
f2386	Ao	10·24 (4)	10·10	10·15	10·27	10·37	10·23
f2317	A3	10·33 (5)		10·27	10·30		10·30
f2382	A2	10·43 (2)		10·31	10·46		10·40
f2401	Ao	10·63 (2)	10·55	10·58	10·72	10·78	10·65

comparison. The first set is Stoy's *Pg'* magnitudes *, taken in preference to his *IPg*, as there is accumulating evidence for a systematic error in the latter for stars fainter than $9^m.0$, although up to this point the *IPg* values are probably of greater accuracy. The *Pg'* scale is really that of *Harvard Annals*, **89**, averaged over all E regions and considerably smoothed. The second set is a group of magnitudes obtained by exposures made with rotating sectors, using the Cape photometric cameras, which are at present attached to the 74-inch mounting in Pretoria. The photographs (in-focus images) were measured both at Pretoria and at the Cape. In reducing the Pretoria measures, magnitudes of stars brighter than $8^m.9$ were assumed from the Fabry results, but this was not done in the Cape measures and the assumed magnitudes are not included in the Table. The third set of magnitudes was obtained also from photographs made at Pretoria, but measured only at the Cape. These consisted of intercomparisons of E2 and E5 with the Præsepe cluster, and then of intercomparisons of E regions, giving finally magnitudes in E3. The Præsepe magnitudes of Haffner † formed the starting-point. Finally there are the Harvard magnitudes which rest on work begun by Miss Leavitt and completed by Miss Payne.‡

The zeros of the Fabry and sector scales are arbitrary, and since we do not wish to give these two preferential treatment, a zero correction $+0^m.04$ has been applied to *Pg'* and $-0^m.13$ to the Harvard magnitudes, in order to make them agree as closely as possible with the rest of the Table. These corrections should not be regarded as of wider significance—that is, as necessarily indicating that the *Pg'* or Harvard magnitudes, taken over all E regions, require any zero correction. Here we are examining only the question of scale, not that of the zero.

Apart from the zero, and the connection between *Pg'* and Harvard magnitudes, noted above, some of these scales have a certain amount of measuring technique in common, but in other respects all five are independent. Each probably has small systematic errors

* *M.N.*, **103**, 297, 1944.
† *Veröff. Univ. Sternwarte, Göttingen*, No. 54, 1937.
‡ *Harvard Annals*, **89**, 1, 1931.

of its own, but a discussion of these must be postponed until more observational material has been accumulated. The general inter-agreement is fairly good. Taking the means of all observations and finding the average deviations from these, we have:

	Number of Stars	Average Deviation
		m
Fabry	17	0·02
Pg′	11	0·04
Sector	17	0·04
Præsepe	17	0·04
Harvard	9	0·07

One star, the faintest of all, has been rejected from the Fabry list. It had only one observation and differed by $0^m.2$ from the mean of sector and Præsepe values.

Fig. 4 gives the Fabry magnitudes plotted first against all others, and then against the mean of *Pg′*, sector and Præsepe. Of the five sets of magnitudes in the Table, those

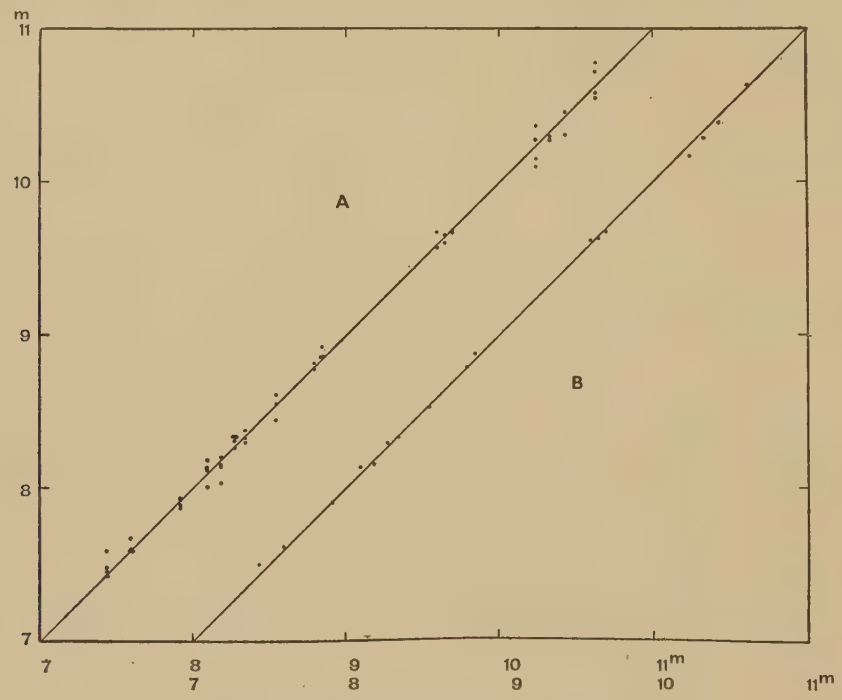

Fig. 4.—Abscissæ: magnitudes derived by the Fabry method.
Ordinates: A magnitudes from four other sources;
B mean magnitudes from three other sources.

obtained by the Fabry method appear to be the most accurate. If, however, we restrict ourselves to stars brighter than $9^m.0$, Stoy's values, both *Pg′* and *IPg*, are equally good.

The writer wishes to express his gratitude to Dr. Stoy and Mr. Cousins for reading through and criticizing an early draft of this paper, and to Dr. Stoy for the use of unpublished magnitude material.

Radcliffe Observatory,
Pretoria:
1945 *August.*

ON THE PHOTOMETRIC USE OF PLATES TAKEN FOR THE DETERMINATION OF STELLAR PARALLAX

R. H. Stoy and A. Cormack

(Communicated by H.M. Astronomer)

(Received 1945 September 24)

Summary

Photographic magnitudes with a probable error $\pm 0^m.05$ have been derived for stars in 150 regions from plates taken in the normal parallax programme at the Cape Observatory. This accuracy has been achieved partly by the quantity of the data used and partly by its careful selection. The magnitudes, which are approximately on the International System, range from the first to the thirteenth magnitude. The scale was determined by a rotating sector while the zero-point was found by comparison with the visual magnitudes of 58 stars of known spectral type. The photographic magnitudes of *Sirius* and *Canopus* found in this way are -1.49 and -0.63; that of Kapteyn's star is 10·39.

Introduction.—The vast store of parallax plates, some photographic, some photovisual, accumulated at various observatories during the past thirty years provides a rich mine of raw material awaiting exploitation. The uniformity of the images of these plates offers an opportunity of tying together the magnitudes of bright and faint stars on a single system with a minimum of complications due to colour. Obviously the first photometric use for these plates should be the determination of the magnitude and colour of the stars whose parallaxes have been determined and of the comparison stars used in this determination. In the derivation of absolute magnitudes, individual magnitudes accurate to less than $0^m.1$ are unnecessary, since limitations in the parallax data introduce uncertainties greater than $0^m.1$ except for the relatively few stars whose parallaxes exceed $0''.1$. Accurate colours, on the other hand, are most desirable, since, for a given spectral type, the colour of a star is intimately connected with its absolute magnitude. Systematically accurate magnitudes and colours for the comparison stars should be useful in computing the correction needed to reduce the observed relative parallaxes to absolute parallaxes.

The colour and magnitude aspect of parallax programmes seem in general to have been much neglected. Thus in 1945 it is possible to publish a list * of physical properties of the 39 stars known to have a parallax of $0''.2$ or over without giving any measures of their colour, while for two of these stars the brightness is given only to the nearest whole magnitude.

Several observatories have got rough magnitudes for the comparison stars from diameter measures of the star images on parallax plates, the magnitude of the parallax star being obtained from some external source. Usually only a single plate has been measured. The McCormick observers †, who have investigated the question more thoroughly than most, calibrated one of the plates for each region with an exposure on the North Polar Sequence. They used a Schilt photometer to measure the plates. The photovisual magnitudes obtained had a probable error of $\pm 0^m.12$, $\pm 0^m.08$ of this being inherent in the plate and $\pm 0^m.09$ being introduced during the reduction of the measures.

* *P.A.S.P.*, 57, 34, 1945. † *Publ. McCormick Obs.*, 7, 3, 1937.

At the Cape, Dr. Jackson has derived rough photographic magnitudes for the first six hundred stars for which the parallaxes were completed. The method he used is described in the introduction of Volume XIV of the *Annals of the Cape Observatory*, where the observational details of the Cape parallax programme can also be found. Jackson had intended to adopt for the parallax star the photographic magnitude given in the Henry Draper Catalogue and to derive the photographic magnitudes of the comparison stars. He found, however, that a more consistent set of magnitudes could be obtained by assuming the mean magnitude of the comparison stars from the time of exposure and the brand of plate used, and deriving the magnitude of the parallax star by allowing for the reduction in brightness effected by the rotating sector. That such a simple method should give results more consistent than the HDC magnitudes may seem surprising to those who are unfamiliar with the way in which these latter were derived from a number of different sources of varying photometric reliability.

Jackson gave no indication of the size of the probable error to be associated with his magnitudes. It is to be noticed, however, that the assumption of a magnitude for the mean of the comparison stars is not based on their appearance on a single plate, but on a whole series of plates. For each region a trial plate having a series of exposures with different settings of the rotating sector and with different exposure times is taken. The comparison stars are selected from this plate, and the time required to give good images of them is estimated. From the same plate the setting of the rotating sector needed to reduce the image of the parallax star to the same appearance as that of the comparison stars is gauged. The suitability of this sector setting and exposure time is checked by the appearance of the images on subsequent plates. With a long focus telescope and with images of the size and density used for parallax work, a difference in magnitude of $0^{m}.1$ is noticeable.

At Dr. Jackson's suggestion we recently used a modified Schilt photometer to extend his magnitude work to the 150 regions in the seventh and eighth lists of Cape Parallaxes published in *Monthly Notices*.* Measurement with the photometer proved to be not only more accurate than the micrometrically measured diameters that Jackson had used, but easier and much faster. A more ambitious procedure could therefore be attempted. The photometer reading for any image was reduced to a preliminary (tabular) magnitude by means of a standard table. This tabular magnitude was then corrected for:

(1) Exposure time differing from the standard exposure time.
(2) The kind of plate used.
(3) Differential atmospheric extinction.

And in the case of the central parallax star for:

(4) The reduction effected by the rotating sector.
(5) The reduction effected by the "5-magnitude" filter if used.

It is to be noticed that our basic assumptions remain the same as Jackson's. These are (*a*) that the magnitude of a star can be gauged from the strength of the image it produces on a given photographic plate in a given time, and (*b*) that the relation between the brightness of a star and the strength of image it produces is the same for all plates. These assumptions are obviously not true in general, so that we must determine with some care the limits within which they are likely to form a useful working hypothesis.

Preliminary Investigations.—The standard table used to reduce the photometer measures to preliminary magnitudes was intended to give without any correction the magnitudes of a star as recorded on an average Ilford Zenith plate having a three-minute exposure. It was based on five plates of the Harvard Standard Regions E2 and E3 specially taken for us by Dr. Jackson. Each of these plates had a series of exposures varying from 15 seconds to 8 minutes, while one of the brighter stars was reduced in

* *M.N.*, 96, 115, 1935; 97, 79, 1936.

brightness by a rotating sector which was changed between the exposures. These five plates thus furnished information about corrections (1) and (4) above in addition to providing the material for the standard reduction table. The magnitudes assumed for stars in E2 and E3 depend mainly on recent work done at the Radcliffe Observatory and at the Cape, though for some of the faintest stars use had to be made of the original Harvard magnitudes suitably corrected.

It is clear that standard tables such as that for reducing photometer measures to magnitudes, or that for allowing for the effect of varying exposure time, must be used with

TABLE I

Difference in Tabular Magnitude produced by Doubling the Exposure

Units $0^m.01$

Tab. Mag.	10	11	12	13
Plate		(*a*) 15–30 secs.		
32966	...	54	67	80
32994	66	59	51	44
Mean	...	57	59	62
		(*b*) 30–60 secs.		
32966	66	68	70	72
32994	64	67	70	73
33070	64	65	68	58
33156	...	73	73	62
Mean	...	68	70	66
		(*c*) 1–2 mins.		
32966	70	70	70	70
32994	75	72	69	66
33070	74	66	57	50
33156	67	65	63	62
33084	58	53	55	56
Mean	69	65	63	61
		(*d*) 2–4 mins.		
33070	60	61	62	64
33156	65	61	56	54
33084	62	67	67	66
Mean	62	63	62	61

Note.—Plate 32966 was taken under variable seeing conditions, plate 33070 under very poor seeing conditions.

discrimination. The basic condition for such tables to be of use is that the curves obtained by plotting photometer readings against magnitudes should all be parallel. How far this is the case for plates taken with the Victoria Telescope can be seen from Table I, which gives the differences in tabular magnitude produced by doubling the exposure in the case of the five experimental plates of E2 and E3. If the photometer reading–magnitude curves had all been strictly parallel, the difference in Table I would be independent of magnitudes. This seems to be true for the mean of a few plates, though for individual exposures there are relatively large deviations, especially in the case of short exposures. The explanation of these deviations is probably to be found in the structure of the star images and its variation with seeing conditions. The use of standard tables should therefore be confined to plates taken under normal seeing con-

ditions; plates taken on nights of very good or very poor seeing conditions should be avoided.

Table I also indicates the sort of correction needed to allow for varying exposure time. It indicates that for modern Ilford Zenith plates the magnitude effect of doubling the exposure is something slightly greater than $0^m.60$, the value found by Dr. Halm * and assumed throughout this work.

The Effect of the Rotating Sector.—The gauges for setting the sector opening were designed on the assumption that the opening had to be reduced in the ratio 1 : 3·16 (anti-log 0·5) to cut down the image by one magnitude. It is now generally accepted that the effect of a fast rotating sector is equivalent to a cutting down of the intensity of the light and not to a cutting down of the exposure time.† The true values of the reduction effected by the various settings of the rotating sector will therefore be 25 per cent. greater than the nominal value. The adopted values shown in the second column of Table II are based on new geometric measurements of the sector openings. Jackson determined the reduction effected by the sector in various ways from assumed magnitudes. The values he adopted, which are shown in column three of Table II, therefore depend on the scale of these magnitudes. If he had applied the corrections determined from the HDC magnitudes together with the 10 per cent. increase indicated by the difference of the Harvard and Cape scales, he would have got the values shown in column four, which agree much better with the values adopted in this paper.

TABLE II

The Rotating Sector Reduction Constants

Nominal Value	Adopted	Jackson	
0·6	0·75	0·6	
1·0	1·26	1·0	
1·5	1·88	1·5	1·74
2·0	2·52	2·1	2·45
2·5	3·16	2·7	3·04
3·0	3·72	3·4	3·68
3·5	4·44	4·0	4·35
4·0	5·02	4·8	4·98
4·5	5·62	5·3	5·42
5·0	5·97	6·0	6·10

The adjustable rotating sector described by Jackson was replaced in 1938 by a series of interchangeable fixed sectors which, being smaller, have the great advantage of permitting the use of comparison stars closer to the parallax star. The computed reduction values for this new set of sectors together with the observed values found from the recent plates of E2 and E3 are shown in Table IIA. The observed values are close to the computed values and show no sign of any significant systematic difference from them.

Observational Material.—From 20 to 30 plates are available for each of the 150 regions to be examined. The plates were exposed between 1933 and 1936 and their serial numbers range from 13,000 to 18,000. Three kinds of plates were used: Ilford Zenith (or Zen.), Ilford Monarch (or Mon.) and Ilford Special Rapid, Extra Sensitive (or ISS). The exposure time for all varieties of plates averaged about three minutes. The ISS plates were used for the regions with the brighter comparison stars, while Zen. or Mon. were used for the remainder. For a number of regions, however, a mixture of plates was used. In most of these cases the exposure time for the ISS plates was about three minutes, while that for the Zen. or Mon. was one minute or just over.

* Introduction to *Magnitudes of Stars contained in the Cape Zone Catalogue of 20,843 Stars*, London, 1927.

† *J. opt. Soc. Amer.*, **23**, 157, 1933.

TABLE IIA

Reduction Constants for the New Rotating Sectors

Nominal Values	Calculated	Observed	Nominal Values	Calculated	Observed
0·3	0·48		2·25	2·86	2·92
0·6	0·76	0·77	2·5	3·14	3·12
0·8	0·98		2·75	3·57	3·53
1·0	1·28		3·0	3·76	3·73
1·25	1·58		3·25	4·30	4·39
1·5	1·90	1·92	3·5	4·39	4·49
1·75	2·14	2·19	3·75	4·78	4·75
2·0	2·50	2·42	4·0	5·26	5·31

The plates to be measured were chosen as far as possible from those to which the parallax measurers had assigned a weight of 0·8, in order to ensure a reasonable uniformity amongst the images to be measured. These images are all moderately exposed—a fairly even grey, fully formed but not at all burnt out. Plates taken on nights when the observer noted cloud or haze were of course rejected.

At first only two plates of each region were measured. On each of these plates all three images of the parallax star and of each of the four or five comparison stars were measured, and the measures reduced to tabular magnitudes with the help of the standard table. A comparison of the tabular magnitudes for the two plates of each parallax region showed that the relative agreement of the comparison stars amongst themselves was all that could be desired (the average difference between the difference for an individual comparison star from the mean of all the comparison stars was only $0^m.04$). On the other hand, the zero point difference between two plates averaged $0^m.25$. Three extra plates for each region were therefore selected and the three images of the parallax star only measured on each. These plates were selected as far as possible so that each of the five plates of each region measured came from a different epoch.

The means of the five tabular magnitudes for each of the parallax stars were taken, and the differences of the individual tabular magnitudes from the mean of the five were arranged according to the type of plate and plate number. As had been suspected, there were definite systematic effects depending on when the plates were taken. The following systematic corrections resulted from this examination:—

			m
For ISS plates numbered	13,000–13,999	Apply	−0·11
	14,000–14,999		+0·33
	15,000–15,499		0·00
	15,500–16,199		−0·17
	16,200–16,999		0·00
	17,000–17,299		−0·17
	17,300–17,999		0·00
	18,000–		−0·13
For Zen. plates numbered	13,000–14,599	Apply	−0·20
	14,600–15,799		0·00
	15,800–16,599		+0·15
	16,600–16,999		+0·08
	17,000–17,999		0·00
	18,000–		−0·26

The application of these corrections reduced the average residual from $0^m.19$ to $0^m.12$ in the case of the ISS plates and from $0^m.17$ to $0^m.13$ in the case of the Zen. plates. Thus

the mean accidental error of the mean tabular magnitude obtained from five plates should be only $0^m.06$, a probable error of under $\pm 0^m.05$.

The mean tabular magnitude of each parallax star was then corrected for the reduction of the rotating sector and for the effect of an exposure time differing from three minutes. Very few exposure times differed greatly from three minutes, so that even if our assumption that doubling the exposure alters the tabular magnitude by $0^m.60$ is not quite correct, the resulting error in the final magnitude is likely to be small.

To determine the zero correction to be applied depending on the kind of plate, use was made of the visual magnitudes of 58 of the parallax stars, which were supplied to us by Dr. Knox-Shaw. These magnitudes are based on Harvard visual observations, but have been reduced to a homogeneous system which does not differ greatly from the International one. They were converted to "photographic" magnitudes by the application of the colour index as deduced from the spectral type, Seares's C_s values being used for this purpose.* The photographic magnitudes so derived were compared with the corresponding tabular magnitudes after they had been corrected for exposure time and sector but not for plate or filter.

The mean differences were as follows:—

					m
For 19	stars	observed	without filter	on ISS plates	+0·59
20	,,	,,	,,	Zen.	−0·52
6	,,	,,	,,	Mon.	+0·42
13	,,	with	,,	ISS	+5·99
15	,,	,,	,,	Zen.	+4·86
2	,,	,,	,,	Mon.	+5·91

These results were combined with the data resulting from stars that had been observed on more than one kind of plate. From 44 stars the mean zero-point difference between ISS and Zen. plates was found to be $1^m.08$, while that between Mon. and Zen. as found from 15 stars was $1^m.00$. The zero-point corrections actually adopted were:

m		
−0·60	for	ISS plates.
−0·50	,,	Mon.
+0·50	,,	Zen.
−5·40	,,	the effect of the filter.

Considering the nature of the data, the residuals from those adopted values are very small and can probably be regarded as a general confirmation of the accuracy of our assumptions. In particular the three independent determinations of the difference in zero-point corrections required by ISS and Zen. plates ($1^m.11$, $1^m.13$ and $1^m.08$) agree fairly well. The plates concerned in the first two of these determinations all have exposure times of about 3 minutes, but for the third determination the average exposure time for the ISS plates was 3 minutes while that for the Zen. plates was only a minute. Consequently this third result should contain to a marked degree the effect of any error in our assumption that doubling the exposure time produces a change of $0^m.60$ in the tabular magnitude. The error in this assumption appears then to be less than 4 per cent. and so in the great majority of cases will have a negligible effect on the final result.

Jackson's value of the reduction constant for the filter as found from the South Polar Sequence was $-5^m.60$.

The correction to compensate for the difference between the atmospheric extinction for the individual stars and that for the mean of the stars for which visual magnitudes were available was based on an assumed zenith extinction of $0^m.48$.† The maximum value of this correction was $0^m.18$, but its average arithmetical value was only $0^m.03$.

* *Ap. J.*, **98**, 276, 1943.

† *M.N.*, **103**, 295, 1943.

The resulting magnitudes for the parallax stars are given in Table III. The first column gives the number of the star in the lists already published in the *Monthly Notices*, the second column the spectral type, the third column the relative parallax as measured at the Cape in units of 0″·001, the fourth column the measured photographic magnitudes, the fifth column the visual magnitudes supplied by Dr. Knox-Shaw, the sixth column the resulting colour index and the seventh column the absolute photographic magnitude calculated from columns three and four. A correction of 0″·002 was applied to the relative parallaxes to make them absolute. No absolute magnitude is given for stars whose relative parallax is less than 0″·01. Where two spectral types are given, the first is the HD type and the second is the revised type given in the Lick General Catalogue of Radial Velocities. The magnitudes and spectral type given for No. 645 in the original list really belong to the brighter companion star, No. 594 of the sixth list. The colour index found for No. 669 is very small for a spectral type of Go.

The magnitudes of the comparison stars were obtained from those adopted for the parallax star by allowing for the difference in magnitudes between the comparison stars and reduced parallax star as measured on two plates of each region.

Discussion of the Results.—We have no information about the colour equation of these photographic magnitudes. This can only be determined by direct comparison with a standard system, a standard system which is at present non-existent in the southern hemisphere. The effect of colour on these magnitudes should not be large, however, since they result from measures of a series of images of uniform size and intensity. In the case of the really bright stars which have been observed through the five-magnitude filter an additional complication is to be feared. The filter is nominally neutral, but it may well have introduced a small reddening effect. The present series of measures is too restricted, however, to evaluate such an effect.

The scale is virtually that defined by the reduction constants of the rotating sector and should be normal and free from colour complications. The zero-point is fixed by the application of mean colour indices to the visual magnitudes of 58 stars. Both colour indices and visual magnitudes are on the International System, so that the resulting photographic magnitudes should also be on this system.

It will be noticed that no attempt has been made to correct the scale of the tabular magnitudes which were derived from Zen. plates when using them for ISS or Mon. plates. This question was investigated by means of stars which had been observed on both ISS and Zen. plates, the differences between the magnitudes as derived from each series of plates being arranged against the tabular magnitudes on the ISS plate. No trace of a systematic difference depending on magnitude could be detected. Due to the care with which the exposure time is chosen to produce an image suitable for measurement, there is only a small range in the tabular magnitude (which averaged about 12·2), so that even if the scale of the reduction tables were systematically wrong the effect on the final magnitudes would be very small.

Incidentally the magnitudes corresponding to a tabular magnitude of 12·2 on a plate with 3-minute exposure are 11·6 for an ISS and 12·7 for a Zen. These magnitudes should correspond to the values assumed by Jackson for the mean magnitudes of the comparison stars. The values he actually adopted were 11·8 and 12·7 respectively.

We do not pretend that the magnitudes that can be got from parallax plates are as good as those that could be got from special observations. Such special observations, however, cannot be reduced until a reasonably reliable system of standard magnitudes has been established in the southern hemisphere. Meanwhile, it seems worth while to use existing observational material to derive with relatively little trouble magnitudes which, if not actually of first-class quality, are far better than anything yet available for the majority of the stars.

Photographic Magnitudes of Sirius and Canopus.—Some of the long series of plates taken to determine the parallax of *Canopus* have numbers lying between 14,000 and 18,000.

TABLE III

No.	Sp.	π	m_{pg}	m_v	C.I.	M_{pg}
601	Go	+ 37	8·23	7·84	+0·39	6·2
602	Fo	+ 78	5·32	5·29	+0·03	4·8
603	F8	+ 38	6·66	6·07	+0·59	4·7
604	...	+ 6	11·74			
605	F5, F5s	+ 44	4·73	4·51	+0·22	3·0
606	...	+ 9	11·46			
607	Ko	+ 82	9·12			8·7
608	...	− 44	13·21			
609	...	+ 11	13·08			8·7
610	Ko, Ko	+ 47	3·94	3·05	+0·89	2·4
611	B1, B1ss	+ 22	1·72	1·97	−0·25	−1·4
612	...	+ 41	9·68			7·9
613	...	+ 10	10·51			5·9
614	K2, K5	+136	7·20	6·52	+0·68	7·7
615	Go	+ 41	9·15			7·3
616	F8	+ 17	7·70			4·1
617	Go	+ 75	6·13	5·80	+0·33	5·5
618	Go	+ 56	7·39	6·92	+0·47	6·2
619	...	− 19	12·83			
620	...	+ 56	12·92			11·7
621	Ao	+ 12	5·19	5·31	−0·12	0·9
622	...	+ 77	11·27			10·8
623	...	+ 15	9·29			5·4
624	Ko, Mo	− 2	10·14			
625	F5, F3	+ 41	5·69	5·30	+0·39	3·9
626	G5	+ 87	7·97			7·7
627	G5	+ 9	7·56	6·85	+0·71	
628	G5, K2	+ 37	7·99			6·0
629	G5, Go	+ 5	7·18	6·65	+0·53	
630	B9	+ 3	8·23			
631	...	− 9	11·31			
632	Go, G2	+ 58	6·19	5·47	+0·72	5·1
633	K2	− 7	8·47	6·79	+1·68	
634	G5	+ 46	10·20			8·6
635	Ko, G9	+ 51	3·37	2·28	+1·09	2·0
636	...	+ 10	11·08			6·5
637	K5	+129	10·60			11·2
638	...	+ 39	13·14			11·2
639	K5	+ 5	10·83			
640	G5	− 2	8·74			
641	G5	+ 23	8·46			5·5
642	Go	+ 4	9·37			
643	..., F4	+ 9	11·72			
644	Go	+ 27	7·38	6·91	+0·47	4·7
645	...	+ 53	8·21			6·9
646	Go	+ 24	7·95			5·0
647	Go, G2	− 7	10·48			
648	Go	+ 21	7·33	6·99	+0·34	4·1
649	G5, G5	+ 18	9·27			5·8
650	Go	0	9·65			

No.	Sp.	π	m_{pg}	m_v	C.I.	M_{pg}
651	...	+ 13	11·08			7·0
652	F5	+ 35	8·18			6·0
653	F5, F8s	+ 58	5·56	5·21	+0·35	4·5
654	G5	+ 45	7·96			6·3
655	F8, F2n	+ 39	5·21			3·3
656	...	+ 60	11·26			10·2
657	Go, Go	+ 31	7·87	7·45	+0·42	5·5
658	Ko, G7	+288	4·01	3·48	+0·53	6·3
659	F8	+ 18	9·38			5·9
660	G5	+ 12	9·74			5·5
661	G5, G1	+ 28	7·20	6·85	+0·35	4·6
662	G5	+ 43	6·36	5·80	+0·56	4·6
663	G5	+ 32	10·14			7·8
664	...	+ 26	13·27			10·5
665	Ko, K5	+ 76	8·67			8·1
666	F8, F6	+ 15	7·11	6·74	+0·37	3·3
667	Ko, G9	+ 97	4·02	3·63	+0·39	4·0
668	Ko, G9	+ 70	4·73	3·79	+0·94	4·0
669	Go	+ 40	8·08	8·05	+0·03	6·2
670	K5, Mo	− 17	6·24	4·80	+1·44	
671	...	+ 23	11·40			8·4
672	F5	+ 22	6·46	6·19	+0·27	3·4
673	A5	+ 10	(7·92)			3·3
674	...	+ 5	10·77			
675	...	+ 30	10·05			7·6
676	...	+ 46	13·27			11·7
677	G	+ 12	10·79			6·5
678	Ko, K5	+ 22	8·53			5·4
679	Go, G2	+ 33	3·52	2·88	+0·64	1·2
680	Fo, cF6	+ 30	2·62	2·65	−0·03	0·2
681	A3	+ 59	3·82	3·91	−0·09	2·8
682	Go	+ 69	8·55	8·33	+0·22	7·8
683	Ko, G5	+ 44	4·50	3·72	+0·78	2·8
684	...	+ 28	13·30			10·7
685	...	+262	12·35			14·5
686	...	+ 16	10·57			6·9
687	...	+ 42	13·13			11·4
688*p*	F2	+ 38	6·46	6·35	+0·11	4·5
688*f*	...	+ 31	7·13	6·98	+0·15	4·7
689	G5	+ 37	7·53	7·11	+0·42	5·5
690	G5	+ 22	8·59			5·5
691	G5	+ 40	7·98			6·1
692	Ko, Ko	+ 4	7·72	6·76	+0·96	
693	Go	+ 27	8·25			5·6
694	Fo, F5p	− 12	6·14	5·96	+0·18	
695	...	− 23	12·88			
696	Ko, Ko	+ 35	5·82	4·84	+0·98	3·7
697	G5	+ 36	8·71			6·6
698	...	+ 25	11·46			8·6
699	K2	+ 28	10·74			8·1
700	Ko	+ 28	9·70			7·1

No.	Sp.	π	m_{pg}	m_v	C.I.	M_{pg}
701	G5	+ 28	10·22			7·6
702	...	+ 54	10·30			9·0
703	K5	− 12	9·62			
704	...	+ 20	9·33			6·0
705	...	+ 16	12·43			8·7
706	G5, K1	+ 11	9·92			5·5
707	K5, K5	+ 28	5·09	3·82	+1·27	2·5
708	{F2, A3} Fop	+ 32	3·88			1·5
709	Ko, Ko	+ 22	6·99	6·35	+0·64	3·9
710	G5	0	10·13			
711	Ko, Ko	+ 13	4·95	4·12	+0·83	0·8
712	F5	− 16	8·64			
713	Go	+ 20	9·08			5·8
714	..., Mo	+ 58	10·81			9·7
715	A5	+ 12	3·92	4·15	−0·23	−0·4
716	...	+ 70	11·26			10·6
717	...	+191	11·59			13·0
718	G5, G5	+ 91	5·91	5·03	+0·88	5·7
719	...	+ 10	11·41			6·8
719	...	− 4	11·78			
720	G5	+ 6	9·63			
721	Go	+ 53	8·39			7·1
722	Ko	+ 7	7·01	6·35	+0·66	
723	...	+ 28	11·75			9·1
724	Go	+ 25	6·90	6·19	+0·71	4·1
725	Ao, Aon	+ 21	2·89	3·16	−0·27	−0·3
726	K5	+ 32	9·83			7·5
727	Go	+ 14	8·93			4·9
728	G5	− 5	8·80			
729	F8, F4	+ 50	5·62	4·99	+0·63	4·2
730	G5, G5	+ 88	5·27	4·68	+0·59	5·0
731	Ko	− 11	10·25			
732	K5, K6	+ 58	9·42			8·3
733	Go, G2	+ 45	7·08	6·47	+0·61	5·4
734	...	+ 1	10·74			
735	Go, Go	+ 16	6·84	6·48	+0·36	3·1
736	K5	+ 37	10·58			8·5
737	..., M5	+146	11·96			12·8
738	...	+ 16	12·80			9·1
739	...	− 6	12·74			
740	Go	+ 8	10·36			
741	...	− 1	12·99			
742	Go	+ 32	7·52	7·00	+0·52	5·2
743	...	− 5	10·27			
744	B5	− 2	8·20	8·34	−0·14	
745	...	− 10	11·46			
746	F2	+ 4	7·72			
747	...	+ 40	11·68			9·8
748	G5, G8	+ 72	7·03	6·28	+0·75	6·4
749	...	+ 1	12·44			
750	F8, F4s	+ 98	4·59	4·22	+0·37	4·6

Nine of these plates were selected, measured and reduced in exactly the same way as those for the 150 stars of Table III. The resulting photographic magnitude for *Canopus* is $-0^m.63$. The probable error of this result as determined from the internal agreement of the nine plates is under $\pm 0^m.05$, but a systematic error of this order is to be feared, because the exposure time of these plates, which were taken through the 5-magnitude filter and a reduction sector of $4^m.5$, averaged only 47 seconds. Consequently the rather large correction of $-1^m.16$ had to be applied to allow for the exposure time not being 3 minutes.

The photographic magnitude of *Sirius* was derived from that of *Canopus* by means of seven plates on which both stars appear. These plates were taken through a 10-magnitude filter and contain three series of exposures, one of *Canopus*, having an exposure time of 2 minutes and a reduction sector of $0^m.6$, and two of *Sirius*, the first having an exposure time of 2 minutes and a reduction sector of $1^m.5$, the second an exposure time of 30 seconds and a reduction sector of $0^m.6$.

The difference between the photographic magnitudes of *Sirius* and *Canopus* found from the longer exposures of *Sirius* was $0^m.88$, while that found from the shorter exposures was $0^m.83$, the average deviation of the individual plates from these results being $0^m.04$ for both long and short exposures. The systematic difference of $0^m.05$ between the long and short exposures is probably to be attributed to the inadequacy of the correction of $1^m.20$ for reducing the 30-second exposures to the equivalent of 2-minute exposures. Assigning slightly greater weight to the longer exposures, we get $0^m.86$ for the difference in photographic magnitude between *Sirius* and *Canopus*. This gives $-1^m.49$ as the photographic magnitude for *Sirius*. No correction for atmospheric extinction is required, since *Sirius* and *Canopus* cross the meridian at the Cape at almost equal zenith distances.

We can derive approximate visual magnitudes for *Sirius* and *Canopus* by applying the colour indices deduced from the spectral type. The Mount Wilson spectral type for *Sirius* is A2s. Assuming the colour of such stars is the same as that of the average HD A1*, the corresponding value of C_s is -0.10. The Lick spectral type for *Canopus* is cF0, and the best we can do is to adopt the value of C_s corresponding to the ordinary HD F0, that is $+0.12$. The magnitudes of *Sirius* and *Canopus* are thus:

	Ptg.	Vis.
Sirius	−1.49	−1.39
Canopus	−0.63	−0.75

The photographic and photovisual magnitudes obtained by King † by extra-focal photometry are:

	Ptg.	Vis.
Sirius	−1.36	−1.40
Canopus	−0.50	−0.85

The visual magnitudes usually quoted are the HR ones of -1.58 and -0.86. Other values derived from the same observational material are:

	Zinner ‡	Payne §	Knox-Shaw
Sirius	−1.30	−1.68	−1.59
Canopus	−0.67	−0.79	−0.89

Photographic Magnitude of Kapteyn's Star.—Two of the recent plates of E2 and E3 also contain a number of exposures centred on Kapteyn's star (CPD $-44°612$, CZC 2725). The photographic magnitude of Kapteyn's star found from these plates is 10.39.

* *Ap. J.*, **98**, 251, 1943.
‡ *Veröff. Remeis-Sternw.*, **II**, 1926.
† *H.A.*, **76**, 107, 1915; **85**, 181, 1928.
§ *Harv. Mim.*, Ser. iii, No. 1, 1938.

Acknowledgments

Our thanks are due to Dr. Jackson for proposing this investigation as a suitable task for a summer vacation and for getting us the experimental plates of E2 and E3. We would also like to thank Miss McDonald for doing much of the routine computing, and Dr. Knox-Shaw for providing us with visual magnitudes for over 350 stars occurring in the Cape Parallax Lists. Dr. Redman and Mr. Cousins have been kind enough to read through the first draft of this paper and to suggest improvements in its presentation.

Royal Observatory,
Cape of Good Hope:
1945 *May* 16.

University,
Cape Town.

THE STRUCTURE OF THE GLOBULAR STAR CLUSTERS

E. Finlay-Freundlich

(Received 1944 December 15)

Summary

Making use of the recently discovered fact that the globular star clusters have wide envelopes of faint stars, and the fact that, if these "atmospheres" are considered, the clusters have well-defined finite radii, the theory is developed that the globular star clusters are settling down from an initially adiabatic state and have reached an intermediate, quasi-stationary state consisting of an isothermal core surrounded by an adiabatic atmosphere.

The fact that the most strongly elliptical cores have the lowest atmospheres is thus explained, even apparently in a numerical agreement with the mathematical theory of isothermal-adiabatic systems; an explanation is also provided for the fact that the ellipticity, frequently shown in the distribution of the bright stars, is restricted to the cores. The theory opens quite new possibilities of determining the masses of globular star clusters from the width of the spectral lines in the spectrum of the bright cores. Preliminary estimates, based on the proper motion measured by van Maanen in a few clusters, indicate very large masses of the clusters.

---

1. *Introduction.*—All attempts to explain the existence of isolated globular star clusters in the vicinity of the galaxy have hitherto failed. These attempts were based on investigations of the distribution of the brightest stars in these clusters. Although they revealed common features which indicated that the structure of the globular clusters is governed by laws similar to those governing the structure of gas spheres, they failed to yield conclusive results. For the star counts on which they were based could not distinguish with certainty between stars of different masses—this impaired all conclusions based on statistical mechanics—and, moreover, as the latest observations have disclosed, these counts did not cover at all the whole extent of the clusters. They overlooked a feature which is probably of decisive importance for an understanding of the structure of the clusters and which has been first disclosed by densitometric measurements of about 70 of 93 globular clusters which surround the galaxy.

The densitometric observations brought the following three facts to light: *

(i) The globular star clusters contain very many intrinsically faint stars which extend far beyond the space occupied by the bright stars hitherto investigated. If D_d denotes the angular diameter of a cluster derived from densitometric measurements and D_p the diameter derived in the usual way from photographic exposures, and if bars indicate mean values, then

$$\bar{D}_d = 3{\cdot}7\bar{D}_p.$$

(ii) The globular star clusters are systems of a definite, finite size. The densitometric observations yield

$$\overline{r.D_d} = 225 \pm 8,$$

---

* H. Shapley and A. R. Sayer, *Proc. Nat. Acad. Sc.*, 21, 593, 1935.

when r, the distance, is measured in kpc.; from this we derive for the mean linear radius of the clusters,

$$\bar{R} = 32{\cdot}8 \text{ pc.} \pm 1{\cdot}2.$$

(iii) The ellipticity shown in the distribution of the bright stars in numerous globular clusters is confined to the central bright core alone. The envelopes of faint stars do not disclose any measurable deviation from sphericity.

These facts open a new approach to an understanding of the nature of the globular star clusters.

2. *Theoretical Considerations.*

(*a*) *General Remarks.*—The main problem presented by the globular star clusters is their very existence as finite systems, isolated from the field of the galactic stars. Their structure must be determined solely by the gravitational field set up by the stars which constitute such a cluster. Whether the influence of the gravitational field of the whole cluster alone or in addition the influence of irregular forces, arising from critical approaches of individual stars, are considered as decisive factors, the final state should correspond in every case to an isothermal distribution.*

This result entails necessarily the complete dispersion of a cluster of finite mass and therefore does not provide us with a practicable solution of the problem under consideration. Actual collisions between stars, if they should happen frequently, would seriously affect the results, because such collisions would not be elastic. But they may be completely disregarded.

We consequently must conclude, since all globular star clusters have apparently well-defined finite diameters, that they have not yet reached the final state of equilibrium. They must be in a quasi-stationary state of sufficient stability to last for a sufficiently long time. On this assumption the theory which follows is based; we shall say that a system is in a quasi-stationary state, if it satisfies the necessary condition of the Virial Theorem: $T/V = -\frac{1}{2}$, where T denotes the kinetic and V the potential energy of the system under consideration.

(*b*) *Quasi-stationary Systems.*—We shall first prove the theorem: a finite spherical system of radius R, consisting of a very large number of gravitating stars the velocity distribution of which satisfies Maxwell's law of distribution, is quasi-stationary when the density ρ and the pressure p satisfy the hydrostatic equation

$$\frac{1}{\rho}\frac{dp}{dr} = -\kappa\frac{M(r)}{r^2},$$

where $M(r)$ denotes the mass included in the sphere of radius $r \leqslant R$ and p is assumed to vanish for $r = R$.

The notions of a pressure p, density ρ and temperature θ similar to the corresponding notions in the dynamical theory of gases remain valid for such a system; p, ρ, θ can be treated as continuous functions of the distance, r, from the centre of the cluster.

We shall consider an element of volume v containing n stars of mass m. If we define the pressure p as the amount of momentum transmitted to the unit of the surface, and the density ρ as the ratio nm/v we obtain the following equations:—

$$pv = \tfrac{1}{3}nm\overline{c^2} = \tfrac{1}{3}\overline{c^2}.\Delta M = \tfrac{2}{3}\Delta E, \quad \text{and} \quad p/\rho = \tfrac{1}{3}\overline{c^2},$$

where ΔM represents the total mass included in the volume v and $\Delta E = \frac{1}{2}\Delta M.\overline{c^2}$ the kinetic energy of the stars included in this volume. The equivalent to the temperature θ

* Charlier, *Medd. Lund Astr. Obs.*, Ser. II, 16, 1917; and E. Martens, *A Research on the Spherical Dynamical Equilibrium-Distribution of Stars of Unequal Masses*, Göteborg, 1928.

has then to be defined by the equation $H\theta = \frac{1}{3}\bar{c^2}$, where H is the universal constant in the kinetic theory of gases. We then derive from the preceding equations:

$$p/\rho = H\theta.$$

These considerations we shall apply to a star cluster *; denoting by dE the kinetic energy included in a concentric shell between two spheres of radius r and $r+dr$ respectively, the total kinetic energy of the cluster becomes $T = \int_0^R dE$.

Since according to the preceding equations:

$$\tfrac{2}{3}dE = \tfrac{1}{3}\bar{c^2}dM = H\theta dM,$$

we obtain

$$T = \int_0^R \tfrac{3}{2}H\theta dM = 6\pi H\int_0^R \theta\rho r^2 dr = 6\pi H\int_0^R \frac{p}{H}r^2 dr,$$

hence

$$T = 6\pi\int_0^R pr^2 dr.$$

Integration by parts yields

$$T = 6\pi[\tfrac{1}{3}pr^3]_0^R - 2\pi\int_0^R r^3 dp.$$

The first term vanishes because $p = 0$ when $r = R$. Thus

$$T = -2\pi\int_0^R r^3 dp.$$

The potential energy, V, on the other hand, is

$$V = -\int_0^R \frac{\kappa M(r)}{r}dM = -4\pi\int_0^R \frac{\kappa M(r)}{r^2}\rho r^3 dr,$$

and since the hydrostatic equation is assumed to be satisfied,

$$-\frac{\kappa . M(r) . \rho}{r^2} = \frac{dp}{dr}.$$

Thus

$$V = 4\pi\int_0^R r^3 dp, \quad \text{and therefore} \quad T/V = -\tfrac{1}{2}.$$

Any density distribution corresponding to that of a polytropic gas-sphere of a class $n < 5$ will satisfy the theorem just proved and similarly any distribution of composite structure, bounded by an envelope built up according to a polytrope of a class $n < 5$; for $n < 5$ polytropic gas spheres are finite. The vast majority of such possible structures will be either too improbable or too unstable to be found realized in nature. An exception is the polytropic system consisting of an isothermal core surrounded by an adiabatic atmosphere; such a structure is from the cosmogonic point of view of special interest. It has been studied in detail by A Ritter †, G. H. Darwin ‡ and R. Emden § in connection with problems concerning condensations of meteoric matter and cosmic dust clouds. With regard to our problem, that of the structure of globular star clusters, it must be expected that an agglomeration of a great number of stars, after having been

* See also R. Emden, *Gaskugeln*, p. 122, 1907.
† Wiedemann's *Annalen*, 16, 1882.
‡ *Phil. Trans. R.S.*, A, 180, 1, 1889.
§ *Loc. cit.*, pp. 183, 237.

violently stirred around by some huge cosmogonic process, will, while settling down, approach such an isothermal-adiabatic structure, before reaching the final and most probable stage of complete dispersion. There are indeed clear indications that the globular star clusters have reached such an intermediate phase. The observational data, it is true, are still quite insufficient to allow a conclusive test of this theory, but the facts that are available support beyond doubt this hypothesis, which at the same time opens various new and interesting possibilities of attacking the intricate problem presented by the globular clusters.*

(*c*) *Isothermal-adiabatic Systems.*—Among the infinite number of possible isothermal-adiabatic systems, according as the temperature of the isothermal core and the total mass of the system is chosen,† one system is of particular interest; it is that of the lowest possible energy content, *i.e.* that of the lowest possible temperature at which an isothermal core can support an atmosphere at all. In this special case the core has its relatively greatest size and the adiabatic atmosphere its relatively smallest extension. We shall find this limiting case realized under exceptional conditions.

Let M_c and D_c denote mass and linear diameter of the isothermal core, M_E and D_E mass and diameter of the whole cluster; the theory of polytropic gas spheres gives the mean square velocity, $\bar{V}^2$, in the core of lowest energy content as:

$$\bar{V}^2 = 1{\cdot}19064 \times 2\kappa M_c/D_c$$

and the following equations between M_c, M_E, D_c and D_E are satisfied:

$$M_E = 2{\cdot}1724 M_c; \qquad D_E = 2{\cdot}7851 D_c.$$

The density ρ of the core as a function of the distance, r, from the centre is derived from a solution of the differential equation:

$$\frac{d^2v}{dr^2} + \frac{2}{r}\frac{dv}{dr} + \beta^2 e^v = 0,$$

where $v = \log \rho$ and β^2 is a constant parameter; the density of the adiabatic atmosphere is derived from the differential equation

$$\frac{d^2u}{dr^2} + \frac{2}{r}\frac{du}{dr} + \alpha^2 u^n = 0,$$

where $u = \rho^{\frac{n+1}{n}}$ and $n = 1{\cdot}5$ corresponds to the polytropic class of the adiabate; α is again a parameter. At the boundary between core and atmosphere the two solutions merge steadily into each other with equal values of density, pressure and temperature.

3. *Application to the Globular Star Clusters.*

(*a*) The only observational data which are available are the diameters D_p defining the space occupied by the giant stars of high luminosity and great masses, and the densitometric diameters D_d defining the extension of the total cluster. The latter values may obviously be identified with the D_E of the "atmosphere", or envelopes of faint stars. The identification of the D_p with the diameters D_c of the isothermal core needs special justification.

On principle there is no means of fixing the boundary between the isothermal core and the adiabatic atmosphere, for they merge continuously into each other. However, two facts support such an identification, at least as a working hypothesis: (1) the process of sorting out of the heavier masses from the lighter masses while the system settled down from the adiabatic state, and (2) the frequently observed ellipticity of the cores.

* It can be shown that, under certain restrictive conditions, every cluster must necessarily pass through a quasi-stationary state before reaching the phase of greatest condensation.

† Emden, *loc. cit.*, pp. 131, 158.

The problem of determining the decrease of the mean value of the mass of the stars with increasing distance from the centre in the isothermal-adiabatic model presents an extremely intricate problem. The following diagram, taken from G. H. Darwin's paper, illustrates the decrease when the masses scatter symmetrically around a mean value between 0 and an upper limit M, according to a frequency function of the form: $f(x) = \eta_0 x(M - x)$.

In the star clusters the masses will range between an upper limit M and a lower limit of about $1/10M$ or $1/20M$; the masses smaller than this lower limit will be without influence upon the structure of the system. We do not have at present any knowledge of the frequency law for the various masses. If we assume a symmetrical distribution around a mean value, following G. H. Darwin, but disregard all masses smaller than

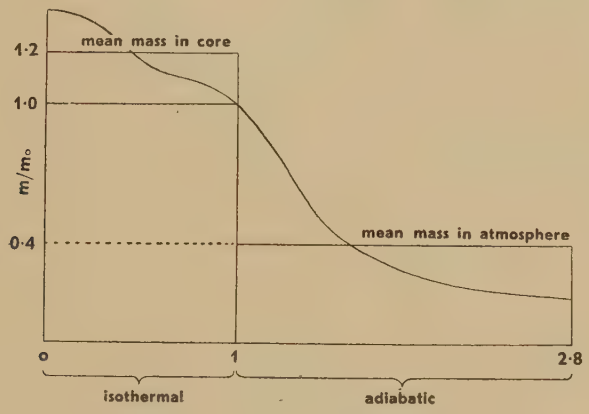

FIG. 1.—*Diminution of mean mass from centre to outside.*

$1/10$ or $1/20$ of the upper limit, M, this entails an error less than 3 per cent. of the total mass. We therefore may derive from the diagram that the mean mass of a star in the core will be about three times the mean mass of the stars constituting the adiabatic envelope. The isothermal core consequently will contain practically all giant stars.

An argument of greater weight in favour of identifying the isothermal core with the space occupied by the bright giant stars is presented by the observed limits in the elliptical distribution of the bright stars.

It has been emphasized before, p. 238, that the envelopes of faint stars remain spherical even when the core shows a pronounced ellipticity. This phenomenon finds a simple explanation within the present hypothesis.

If in the initial adiabatic state of the whole agglomeration of stars the system possessed an angular momentum of rotation and if, while the heavy masses sank down into the core they were unable to transmit their momentum to other stars due to the only very rare occurrence of collisions or of critical approaches, these massive stars must become more and more elliptically distributed the more they approach the centre of the cluster. Thus the fact that the cores only show ellipticity supports strongly the view that they developed from an initially adiabatic distribution. Moreover, there is a distinct relationship between the size of the cores of bright stars and the degree of their ellipticity. This justifies us in identifying the observed D_p with the D_c of the isothermal core in the isothermal-adiabatic model.

The mean values of the ratio of D_E/D_c range from 7·2 to 2·6, the latter value, very near the theoretical limit 2·8, is reached in those cases when the ellipticity of the core is most pronounced. It is evident that this is to be expected, if the equilibrium between core and atmosphere corresponds to that in an isothermal-adiabatic system. The centrifugal force increasing in strength with increasing condensation of the massive

bright stars will tend to counterbalance any further compression of the core by the weight of the overlying stars. Thus systems showing the greatest possible core—of the lowest possible temperature—for which the ratio D_E/D_c should be equal to 2·8, will be found among the clusters with greatest ellipticity of the core. This conclusion is fully confirmed by the observational data which are summarized in the table below.

The ellipticities of the cores are measured, following H. Shapley, by the ratio of the minor to the major axis of the core multiplied by ten. The true ellipticities will be on the average stronger than the observed values because the different possible orientations of the axis of rotation will tend to reduce the observed values. But it is at present impossible to make any assumptions concerning the orientation of the axis in space, and corrections to account for this systematic error have not been applied. The globular clusters belonging to the classes I–IV in Shapley's classification have been discussed separately; they are the most strongly condensed clusters and should disclose the essential features of this theory particularly clearly because their diameters are determinable with great accuracy. The column headed $\bar{D}_E$ in the table gives the mean angular values of the D_E for each group, after having reduced the observed values to one and the same distance—$6{\cdot}8 \times 10^3$ pc. These values which scatter only very moderately are given to show that the pronounced decrease of the ratio D_E/D_c from 7·2 to 2·6 is only due to the increase of the size of the cores. N is the number of clusters used in every group.

TABLE I

	Class	$\overline{D_E/D_c}$	$\bar{D}_E$	N
No ellipticity	I–IV	7·1	34′·8	6
No ellipticity	I–XII	5·3	33·6	17
Ellipticity > 8 :	I–IV	3·4	34·9	8
Ellipticity $\leqslant 8$	I–XII	3·0	31·4	16
The five most strongly elliptical clusters	~	2·6 ± 0·3	37·6 ± 1·2	5

The individual values for D_E/D_c in the last line are: M3 : 2·2; M5 : 2·0; M19 : 3·4; M22 : 2·0; M62 : 3·3. From the discussion, which included 64 clusters, I excluded ω Centauri, a quite exceptionally large cluster which shows already in the bright central core traces of a complex structure *, NGC 4147 and 6388, the ellipticity of which is uncertain, and NGC 104, 6356, 6864, described by Shapley as being abnormally large.

(*b*) *The Probable Masses of the Globular Star Clusters.*—The model for the globular clusters here proposed permits to derive their total masses from a simple formula in those cases where the clusters correspond to the limiting case of lowest energy content. In this case the theory of polytropic systems gives (see p. 240)

$$\bar{V}^2 = 1{\cdot}19064\ \kappa . M_c/R_c,$$

where R_c denotes the linear radius of the core, and $M_E = 2{\cdot}17 M_c$. The observations yield $R_c = 32{\cdot}8$ pc., see p. 238. Values of $\bar{V}^2$ could be easily determined by measuring the width of the spectral lines in the spectra of the bright cores of globular clusters. At present no such observations are available. Only for three globular clusters have the proper motions of stars been measured by Van Maanen.† These values have been used to derive the mean spatial velocities in these clusters; they are given in the second table. The observed proper motions μ'' were changed into the mean tangential velocities with the help of the distances, r, of the clusters and these by multiplication by $4/\pi$ changed into mean spatial velocities.

* J. Shilt, *A.J.*, 889, 1928.
† *Mt. Wilson Contr.*, 338, 1917.

TABLE II

	μ''	N	r	$\bar{V}_{tg}$	$\bar{V}_{spat}$	D_E/D_c	Class
	"		pc.	km./sec.	km./sec.		
M13	$3{\cdot}8.10^{-3}$	62	$10{\cdot}3.10^3$	190	242	1·8	V
M56	$4{\cdot}4.10^{-3}$	50	$20{\cdot}3.10^3$	150	573	4·0	X
M2	$3{\cdot}5.10^{-3}$	76	$13{\cdot}9.10^3$	280	312	2·1	II

Although the accuracy of the observations probably does not guarantee more than the order of magnitude, it may be worth pointing out that for M56, the core of which is relatively half as large as the cores in the two other clusters, the mean velocity in the core is indeed found to be considerably larger than in the other two cases, as it should be. For M13 and M2, which both show ellipticity and seem to correspond to the limiting case of lowest energy, since D_E/D_c are in both cases very small, the mean value of $\bar{V}^2$ would be equal to (280 km./sec.)^2. If we chose this value to determine the masses we obtain:

$$M_c = 3{\cdot}6.10^{41} \text{ gm.} = 1{\cdot}8.10^8\ M_\odot$$
$$M_E = 2{\cdot}17\ M_c \quad = 3{\cdot}9.10^8\ M_\odot$$

These values are considerably larger than all values hitherto assumed for the masses of globular star clusters. Since the velocity enters into the formula in the second power, the value of M_c depends very sensitively on the assumed value of $\bar{V}^2$. Should $\bar{V}^2$ be only of the order of 100 km./sec., M_E would be reduced to about 5×10^7 solar masses; and the total mass of the 93 globular clusters which are attached to the galaxy would become considerably smaller than the total mass of the galaxy, which is estimated as being of the order of 10^{11} solar masses. There are strong reasons, however, to believe that this is not the case. The fact that the globular clusters have been able to maintain themselves separated from the galaxy indicates that the gravitational field set up by them is not negligibly weak compared with that of the galaxy itself.

But there is another observational fact, hitherto not utilized, which may corroborate the large values of the masses obtained for the globular clusters. Although the following argument is still very uncertain it is worth mentioning, since it may open a new and promising way for further investigations.

The few cluster-type variables that have been discovered outside of globular clusters in the galaxy are distinguished by extraordinary large spatial velocities. I believe it to be more likely that these large velocities have to be interpreted as velocities of escape from a globular cluster to which they primarily belonged than, as Oort believes, as velocities of escape from the galaxy. For Oort remarks *: "The high velocities found seem to indicate that the velocity of escape from a point near the Sun would be at least 400 km./sec.".

The velocity of escape from a spherical cluster of about 30 pc. radius having a total mass equal to $4 \times 10^8\ M_\odot$ would be equal to about 300 km./sec. The observed spatial velocities of the four cluster type variables in the galaxy are:

Cluster type variable	SH Aqil	XZ Cygn	RV Capr	Ci. 2018/19
Space velocity (km./sec.)	398	395	331	555

A closer agreement is at present not to be expected. A systematic search for cluster type variables in the neighbourhood of globular star clusters could furnish support of this view and would provide valuable data for an estimate of the masses of the clusters.

Concluding Remarks.—The preceding estimate of the masses of globular star clusters is restricted to the limiting case when the isothermal core has its greatest relative size. For other systems, for which the ratio of D_E/D_c exceeds the value 2·8, no simple formula connects $\bar{V}^2$ with the mass and radius of the cluster. But as soon as observations of values of $\bar{V}^2$ are at hand, numerical integrations of the two differential equations for the isothermal core and for the adiabatic atmosphere (p. 240) will make it possible to test whether the observed values of D_E/D_c correspond to the theory here expounded.

* *B.A.N.*, 318, 1939.

THE VAPOUR PRESSURES OF PLANETARY CONSTITUENTS AT LOW TEMPERATURES, AND THEIR BEARING ON THE QUESTION OF THE ORIGIN OF THE PLANETS

Alfred Lauck Parson

(Communicated by H. Jeffreys, F.R.S.)

(Received 1944 December 17)

Summary

An inquiry into the possibility of planetary nuclei (which could not have originated gravitationally) having been formed by change-of-state condensation from gases shows that this could have occurred under all reasonably conceivable conditions of temperature and density.

In an interesting letter to *Nature* recently * on the "Origin of the Solar System", Dr. Harold Jeffreys, in continuing his perhaps valid contention that the Sun, at the time of the condensation of the planets, could not have been much more distended than it is at present, speaks of the particular importance "in the present state of the subject" of having "some idea of the vapour pressures of ordinary solids at temperatures far below their boiling-points", since the planets could scarcely have commenced condensation from the gaseous state by gravity alone. He added that he had not been able to find data on the subject.

This reminded me of how in the past, during researches on theist cosmology, I had made rough estimates of such vapour pressures for iron and calcium oxide, getting for iron 10^{-44} gm./c.c. at 273° A. (10^{-29} at 400° A.) and a more problematical value for calcium oxide of 10^{-113} gm./c.c. at 273° A. (10^{-29} at 1400° A.)—using the "Trouton rule" and the "Clausius-Clapeyron equation", and assuming a latent heat not varying with temperature.

Now the "Clausius-Clapeyron equation" ($d \log p/dT = L/RT^2$), used over any considerable range, properly requires the variation of the molecular latent heat (L) to be represented, and this is a little difficult seeing that the molecular specific heat of iron (B.P. 2723° A.) is not known experimentally above 1873° A.; but it is recognised from analogies that it cannot increase much above that temperature. The molecular specific heat, about 6 cal. mol.$^{-1}$ deg.$^{-1}$ at 273° A. and 6·55 at 373°, increases to 8·38 at 673° and is that still in the liquid state at 1873°, but with several changes (mostly increases to about 10 or even 16) near and at various transition points. Inspection of the area under the temperature curve shows, however, that the assumption of a constant value of 8·38 over the whole range to be considered gives an adequate "smoothing out" of the curve, especially as we are not much concerned with the small range below 100° to 400° A.

Now from "Trouton's rule", that the molecular latent heat at boiling-point of an unassociated substance is $21T_b$, the 21 holding equally for substances such as benzene and mercury, we derive $2723 \times 21 = 57183$ cal. mol.$^{-1}$ as the molecular latent heat of iron at boiling-point. Then, since the molecular specific heat of the monatomic gas is virtually constant at 5·08 cal. mol.$^{-1}$ deg.$^{-1}$, the molecular latent heat at any temperature

* H. Jeffreys, *Nature*, **153**, 140, 1944.

T will be $57183 + (8{\cdot}38 - 5{\cdot}08)(2723 - T)$ approximately; so that the "Clausius-Clapeyron equation", as required here, takes the more accurate form:

$$d \log p/dT = \{57183 + (8{\cdot}38 - 5{\cdot}08)(2723 - T)\}/RT^2.$$

Integrating, we have:

$$\log (p_{2723}/p_T) = R^{-1}[-66169T^{-1} - 3{\cdot}3 \log T]_T^{T=2723},$$

whence, at 273° A., for example, $p = 10^{-46{\cdot}7}$ atmospheres, corresponding to a density of $10^{-49{\cdot}3}$ gm./c.c. Surveying other temperatures, we have for

$T = 273°$ A.	Sat. Dens. $= 10^{-49{\cdot}3}$
400°	$10^{-32{\cdot}9}$
450°	10^{-29}
770°	10^{-16}.

In the letter referred to Dr. Jeffreys considers principally a general density of 10^{-15} (although at the Earth's distance this might have been more like 10^{-11}). I have chosen 10^{-16} for iron in that case, because Bowen's estimate of the present abundance of iron atoms in space is less than some 0·001 of that of hydrogen atoms, but the original abundance must have been relatively greater. The 10^{-29} is a value for iron at the present time, judging from Dunham's and Struve's general results.*

As regards the reliability of these estimates, an error in the Trouton quotient of 5 per cent. (which is not likely) would make only a 4 per cent. error in the index, and a 15 per cent. error in modifying the latent heat (it cannot be more) only a 2 per cent. error in that index. So that we can be reasonably certain that the indices have less than 10 per cent. error.

Now, since iron is the least easily condensed of the substances most important in planetary beginnings (others being CaO, MgO, SiO_2), the conclusion is clear, not only that nuclei could have condensed from vapour in the beginning, but that such condensation must be taking place still. The latter can account for the nearly pure particles of iron in meteoric dust.

So much for iron and still more readily condensed substances. But less easily condensed substances, such as water, ammonia, carbon dioxide, methane, must be the main constituents of the outer planets; although with iron and the rest omnipresent there is no need to suppose that they were required to form the original nuclei. A study of the vapour pressure of water (or rather ice) shows, moreover, that, whatever may, and certainly does, take place at the surface of a body already large, ice can scarcely have been deposited initially in the regions of Jupiter and (probably) Saturn, even at temperatures no higher than the present: ammonia, carbon dioxide and methane would be still less able to condense without gravitational assistance, not succeeding at any distance less perhaps than that of Pluto.

Washburn † has made the most recent and careful estimates of the vapour pressure of ice, down to some −90° C., and extrapolation with his formula, as well as with another devised by the author, and the use of the Clausius-Clapeyron equation as it was used for iron, agree in giving vapour density values for ice of closely 10^{-11} gm./c.c. at 160° A., and some 10^{-15} to 10^{-18} (probably less then 10^{-16}) at 110° A. Thus a water-vapour density of some 10^{-16} seems too low even for the present temperature near Saturn (some 123° A.).

Hillcroft (Allonby),
Mawbray Road,
Maryport,
Cumberland:
1944 *December* 17.

* See C. S. Beals on "Interstellar Matter", *M.N.*, **102**, 96–106, 1942.

† *International Critical Tables*, V, 93, 1929.

STELLAR PARALLAXES DETERMINED PHOTOGRAPHICALLY AT THE CAPE OBSERVATORY (FIFTEENTH LIST)

(Communicated by H.M. Astronomer)

(Received 1945 July 4)

This list is in the same form as previous lists commencing in *M.N.*, **89**, 497 (1929 March).

Each result generally depends on rather more than 30 plates taken over a period of 4 years or more. The results for a few stars previously published are revised with the help of additional plates—generally more than the number taken for the original determination.

For a few stars for which the spectral type is unknown, the colour class as determined by Luyten is given in the column headed Type.

Notes

1310. This star has a faint component south preceding.

1314. A faint companion precedes, and is frequently fused with principal star.

342. Old series of measures with 4 recent plates give $\pi = +''\cdot054$, $\mu = -''\cdot471 \pm''\cdot002$. A new reference system with the new series of plates and six of the best of the old series gave $\pi = +''\cdot045 \pm''\cdot008$, $\mu = -''\cdot461 \pm''\cdot001$. Adopt mean given in table.

1317. Double.

1343. Two stars. The mean of the two components gives p.e. (unit weight) $= \pm''\cdot023$ $\pi = +''\cdot014 \pm''\cdot007$, $\mu = -''\cdot094 \pm''\cdot003$. $\Delta\alpha = 5''\cdot930$ for 1942·0.

1349. Faint companion south following.

1370. Two stars $\Delta\alpha = 2''\cdot00$ for 1941·0. The second star is photographically fainter.

1373–1375. The separation of the components of Σ1998 in R.A. is $11''\cdot286$. The mean parallax of these two stars $+''\cdot030 \pm''\cdot005$. If ξ^1 Scorpii belongs to the same system the mean parallax is $+''\cdot033 \pm''\cdot004$ with the proper motion in R.A. $-''\cdot049 \pm''\cdot003$. Independent plates were taken for the two components of Σ1998 but the two stars were measured on both series of plates, half weight being given to the plates taken with the shutter for the other star. The difference in magnitude is about 0·6.

No.	Name and DM No.	Mag. Vis. Phot.	Type	R.A., Dec. (1900·0)	Unit 0·″001				
					Proper Motion		P.E. Unit Weight	Parallax	P.E.
					R.A.	Dec.			
				h m s					
1301	LPM 1	8·7	K0	0 00 16	− 180	−510	±32	+ 53	± 9
	−68° 3597	9·7		−68° 22′·4	− 148				± 3
1302	GC 789	Var.	F8	0 35 02	+ 325	−111	±36	+ 16	±10
	Z Sculptoris	6·3–7·6		−34° 30′·5	+ 413				± 5
1303	LPM 37	...	m	0 37 25	+ 620	−330	±35	+ 37	±11
	Ci 20, 47	13·0		−52° 54′·0	+ 618				± 5
1304	GC 897	6·70	G5	0 40 24	+ 145	−745	±22	+ 41	± 6
	−66° 53	7·3		−66° 10′·6	+ 159				± 3
1305	GC 925	8·5	F8	0 41 23	+ 248	− 26	±25	+ 30	± 7
	−44° 198	8·9		−44° 39′·0	+ 215				± 3
1306	FS 410	...	...	1 25 06	+ 192	+ 99	±28	+ 15	± 9
	CAZ −49°, 16	13·3		−48° 41′·6	+ 189				± 6

No.	Name and DM No.	Mag. Vis. Phot.	Type	R.A., Dec. (1900·0)	Unit 0″·001 Proper Motion R.A.	Proper Motion Dec.	P.E. Unit Weight	Parallax	P.E.
				h m s					
1307	FS 445	...	m	1 33 12	+ 488	+128	±39	+ 31	±11
	LPM 77	12·1		−49° 42′·5	+ 469				± 6
852	ϵ Eridani	3·31	Ko	3 28 13	− 975	+ 22	±27	+306	± 7
	PGC 814	4·81		−9° 47′·8	−1006				± 2
131	ZC 1743	9·1	K5	3 31 51	+ 394	+330	±31	+102	± 7
	−48° 1011	9·6		−48° 45′·6	+ 385				± 1
1308	FS 1099	...	...	3 46 19	+ 170	−273	±33	+ 23	±10
	CAZ −46°, 76	12·2		−46° 28′·6	+ 175				± 6
137	−36° 1508	7·56	F8	3 51 03	+ 153	+363	±24	+ 21	± 5
		8·0		−36° 00′·1	+ 104				± 1
1309	FS 1140	...	...	3 54 14	+ 250	− 3	±28	+ 6	± 8
	−46° 1239	12·0		−46° 38′·2	+ 240				± 5
1310	FS 1218	...	...	4 07 04	− 66	+285	±33	+ 15	±10
	−50° 1287	12·5		−50° 16′·1	− 58				± 6
1311	FS 1280	...	...	4 17 52	+ 96	−279	±34	− 4	± 9
	−40° 1349	12·4	...	−40° 35′·2	+ 95				± 5
676	Luyten 131–6	...	m	4 54 35	+ 920	−600	±36	+ 39	± 6
	LPM 199	13·27		−61° 18′·2	+ 906				± 2
1312	FS 1597	...	Ao	5 02 46	+ 66	−132	±33	− 7	±10
	−48° 1630	10·8		−48° 01′·2	+ 72				± 5
1313	FS 1609	...	K5	5 03 59	+ 26	+106	±28	+ 13	± 8
	−41° 1715	11·9		−41° 30′·7	+ 34				± 4
	FS 1610	...	K5	5 04 01	+ 22	+ 91	±22	+ 2	± 6
	−41° 1716	11·9		−41° 30′·2	+ 46				± 3
	Mean	...	...	...	+ 24	+ 99	...	+ 7	± 5
					+ 40				± 3
1314	FS 1751	...	K2	5 23 31	− 172	0	±37	+ 18	±11
	−42° 1943	11·8		−42° 25′·4	− 170				± 6
1315	λ Columbæ	4·89	B5	5 49 29	− 10	+ 33	±36	+ 30	± 9
	PGC 1467	4·77		−33° 49′·4	+ 3				± 6
1316	ϵ Doradus	5·15	B5	5 50 00	− 19	+ 13	±27	+ 7	± 7
	PGC 1469	5·03		−66° 55′·6	− 39				± 6
342	LPM 226	7·85	K5	5 56 36	− 410	+370	±30	+ 48	± 6
	−31° 2902	9·2		−31° 02′·7	− 464				± 1
1317	GC 7731	6·44	G5	6 02 11	− 100	− 31	±35	+ 14	± 9
	−48° 2124	7·1		−48° 26′·9	− 90				± 6
1318	ZC 3382	9·3	Ko	6 08 21	+ 78	−273	±30	+ 41	± 8
	−41° 2266	9·7		−41° 32′·8	+ 70				± 4
1319	ZC 3422	8·8	Ko	6 11 43	+ 125	−228	±32	+ 13	± 8
	−49° 2129	9·1		−49° 46′·7	+ 98				± 5

No.	Name and DM No.	Mag. Vis. Phot.	Type	R.A., Dec. (1900·0)	Unit 0.″001 Proper Motion R.A.	Proper Motion Dec.	P.E. Unit Weight	Parallax	P.E.
				h m s					
1320	ζ Canis Maj.	3·10	B3	6 16 28	+ 3	+ 2	±30	0	± 8
	PGC 1601	2·93		−30° 01′·1	+ 6				± 6
1321	λ Canis Maj.	4·48	B5	6 24 28	− 24	+ 23	±28	+ 10	± 8
	PGC 1641	4·36		−32° 31′·0	− 15				± 5
1322	FS 2344	...	Ko	6 34 39	+ 178	− 4	±22	+ 67	± 6
	−49° 2340	11·4		−49° 57′·1	+ 176				± 4
353	GC 9185	6·89	F5	6 55 28	− 15	−372	±28	+ 24	± 6
	−52° 1042	7·8		−52° 30′·1	+ 3				± 2
1323	FS 2763	...	K	7 03 25	+ 135	− 141	±36	+ 37	± 9
	−47° 2804	11·5		−47° 25′·7	+ 144				± 7
1324	FS 2996	...	...	7 17 30	− 80	+169	±36	+ 7	± 9
	CAZ −51°, 194	12·7		−51° 16′·8	− 99				± 6
1325	FS 3280	...	...	7 35 33	+ 135	− 78	±31	+ 1	± 8
	−40° 3289	10·7		−41° 03′·0	+ 113				± 4
1326	FS 3370	...	Ko	7 42 04	+ 65	−121	±22	+ 1	± 6
	−46° 3395	11·3		−46° 09′·7	+ 76				± 3
1327	GC 10819	9·3	G5	7 55 42	+ 11	+255	±30	+ 11	± 9
	−41° 3606	9·2		−41° 19′·3	+ 31				± 5
1328	ZC 5152	9·1	G5	7 56 21	− 146	+233	±29	+ 22	± 7
	−42° 3764	8·7		−43° 01′·9	− 129				± 3
1329	FS 3793	...	...	8 03 05	− 51	+114	±28	+ 4	± 7
	−44° 4044	10·8		−44° 29′·7	− 41				± 5
1330	FS 3936	...	Ko	8 08 49	− 80	+ 25	±35	+ 10	± 9
	−49° 3404	10·6		−49° 57′·2	− 54				± 5
1331	ZC 5777	8·5	Go	8 17 11	− 127	+ 74	±22	− 3	± 7
	−50° 3271	8·7		−50° 34′·2	− 130				± 4
1332	FS 4237	9·2	Ko	8 20 33	− 63	+104	±30	+ 1	± 8
	−51° 2985	9·6		−51° 37′·0	− 29				± 4
1333	FS 4343	9·8	K2	8 24 13	+ 52	− 85	±32	+ 10	± 9
	−46° 4184	10·1		−46° 31′·3	+ 56				± 6
1334	FS 4556	...	...	8 32 41	− 236	+123	±23	+ 28	± 7
	−41° 4300	12·8		−41° 12′·8	− 233				± 3
1335	ZC 6225	9·3	G5	8 33 41	− 73	+166	±28	+ 23	± 8
	−40° 4454	9·4		−40° 58′·4	− 73				± 4
1336	ZC 6453	7·0	Go	8 41 24	− 185	+145	±23	+ 30	± 6
	−44° 4771	7·0		−44° 10′·9	− 187				± 3
1337	FS 4949	...	Go	8 45 42	− 178	+ 96	±34	+ 18	±10
	−41° 4542	10·8		−41° 19′·7	− 179				± 5
1338	FS 5048	...	...	8 47 47	− 201	+ 76	±27	+ 15	± 7
	−43° 3058 *	11·9		−43° 52′·4	− 186				± 4

1338 * CPD Number; not in Co.D.

No.	Name and DM No.	Mag. Vis. Phot.	Type	R.A., Dec. (1900·0)	Unit 0″·001 Proper Motion R.A.	Proper Motion Dec.	P.E. Unit Weight	Parallax	P.E.
				h m s					
1339	λ Velorum	2·22	K5	9 04 19	− 25	+ 7	±32	+ 17	± 8
	PGC 2452	5·3		−43° 01′·7	− 25				± 5
1340	FS 5661	...	G	9 07 14	− 223	+131	±33	− 1	± 9
	−49° 4182	11·5		−49° 21′·8	− 202				± 5
1341	κ Velorum	2·63	B3	9 19 01	− 12	+ 1	±21	+ 10	± 6
	PGC 2526	2·46		−54° 35′·0	+ 1				± 4
1342	LPM 314	...	k	9 20 23	+ 650	−550	±28	+ 18	± 9
	Ross 436	13·8		−6° 53′·6	+ 608				± 4
1343	ZC 7591 p.	9·2	F8	9 32 55	− 113	+ 48	±28	+ 38	± 8
	−47° 5063	9·3		−47° 34′·7	− 92				± 4
	ZC 7591 f.	...	...	...	...	...	±39	− 10	±11
	−47° 5063	...			− 96				± 5
1344	ZC 7593	8·5	G0	9 32 55	− 171	+ 40	±27	+ 20	± 7
	−45° 5285	9·0		−45° 53′·5	− 153				± 4
1345	FS 6696	...	K0	9 45 34	− 85	+102	±26	+ 25	± 7
	−41° 5375	10·6		−41° 42′·1	− 73				± 4
1346	FS 6992	9·9	K0	9 54 47	+ 75	− 62	±33	+ 9	± 9
	−47° 5410	11·1		−47° 34′·6	+ 89				± 5
1347	FS 7138	...	K0	9 59 18	+ 115	− 6	±29	− 2	± 9
	−48° 5251	11·2		−48° 09′·6	+ 116				± 6
1348	FS 7198	9·4	G5	10 01 12	− 175	+ 55	±28	− 2	± 7
	−43° 5900	10·3		−43° 25′·9	− 142				± 4
1349	ZC 8303	8·4	K0	10 08 57	− 228	+147	±33	+ 63	± 9
	−46° 5923	8·2		−46° 59′·1	− 211				± 6
1350	FS 7933	...	...	10 28 24	+ 105	− 5	±38	+ 39	±11
	−40° 6102	11·7		−40° 56′·4	+ 88				± 6
1351	FS 8075	...	G0	10 35 24	− 102	− 20	±29	− 9	± 9
	−46° 6314	10·6		−46° 53′·9	− 96				± 5
1352	ZC 8945	7·5	F2	10 44 16	− 170	+ 37	±27	+ 14	± 7
	−47° 6217	7·7		−47° 49′·4	− 139				± 4
1353	FS 8305	9·9	F8	10 44 39	− 141	+ 28	±24	+ 22	± 7
	−45° 6415	10·0		−46° 04′·0	− 130				± 4
1354	FS 8354	...	...	10 46 25	− 154	− 57	±25	+ 5	± 8
	−44° 6849	12·2		−44° 30′·2	− 143				± 3
1355	FS 8480	...	...	10 54 10	− 226	+ 96	±26	+ 13	± 8
	−42° 6627	12·9		−42° 08′·2	− 238				± 5
1356	FS 8481	...	K0	10 54 20	− 173	− 88	±29	+ 20	± 8
	−46° 6608	11·6		−46° 41′·1	− 174				± 5
1357	GC 15481	7·5	F5	11 10 50	− 191	+ 88	±30	+ 15	± 7
	−41° 6438	8·0		−42° 04′·2	− 186				± 4

No.	Name and DM No.	Mag. Vis. Phot.	Type	R.A., Dec. (1900·0)	Unit 0·″001 Proper Motion R.A.	Dec.	P.E. Unit Weight	Parallax	P.E.
				h m s					
1358	FS 8860	10·0	G5	11 17 35	− 187	+ 94	±33	+ 14	± 8
	−42° 6925	9·8		−42° 46′·7	− 160				± 5
1359	ZC 9617	8·6	Go	11 28 46	+ 165	−161	±33	+ 17	± 9
	−45° 7084	9·3		−45° 45′·7	+ 149				± 5
1360	FS 9720	9·4	K2	12 06 09	− 241	− 20	±25	+ 24	± 7
	−44° 7848	9·8		−44° 18′·3	− 235				± 4
1361	ZC 10217	9·2	Go	12 10 24	− 299	+110	±31	+ 14	± 8
	−47° 7490	9·2		−47° 51′·8	− 272				± 5
1362	FS 10138	...	Mo	12 31 21	− 98	− 82	±26	+ 31	± 7
	−41° 7290	11·9		−41° 09′·8	− 88				± 4
1363	Ross 977	8·3	G5	13 03 15	− 370	+ 70	±30	+ 29	± 8
	−6° 3742	9·1		−6° 46′·6	− 214				± 4
1364	FS 11114	...	Mo	13 26 00	− 104	− 70	±26	+ 11	± 7
	−45° 8522	11·1		−45° 48′·9	− 86				± 4
1365	GC 18826	5·88	Ko	13 50 07	− 151	− 76	±30	+ 21	± 7
	−45° 8815	7·3		−46° 06′·0	− 125				± 4
1366	ZC 12033	9·7	Ko	14 01 47	− 208	+106	±26	+ 19	± 6
	−51° 7976	9·7		−51° 48′·8	− 217				± 4
1367	FS 11947	11·0	G5	14 09 01	− 154	−160	±27	+ 24	± 6
	−50° 8390	10·0		−50° 24′·8	− 154				± 5
1368	GC 19693	7·75	Go	14 31 43	− 349	+ 12	±26	+ 21	± 7
	−3° 3648	8·31		−3° 50′·7	− 303				± 4
1369	GC 19727	6·47	F2	14 32 43	− 184	−125	±30	+ 5	± 8
	−48° 9218	7·3		−48° 37′·2	− 184				± 4
1370	ZC 13079	9·6	Go	15 07 27	− 272	−257	±31	+ 24	± 7
	−40° 9409	9·3		−41° 04′·9	− 248				± 5
	ZC 13080	9·6	Go	15 07 27	− 275	−246	±26	+ 16	± 6
	−40° 9410	9·2		−41° 04′·6	− 238				± 4
	Mean	...	...	...	− 274	−252	...	+ 20	± 5
					− 243				± 3
1371	FS 13258	...	G5	15 23 52	− 210	+ 34	±26	+ 23	± 8
	−47° 7173 *	11·8		−47° 38′·6	− 208				± 5
1372	FS 13662	...	M	15 34 44	− 141	−190	±29	+ 19	± 7
	−50° 9730	12·4		−50° 34′·8	− 124				± 5
1373	ξ^1 Scorpii	4·16	F8	15 58 52	− 65	− 36	±30	+ 38	± 8
	PGC 4082	4·66		−11° 05′·8	− 59				± 5
1374	Σ1998 n.p.	7·4	Ko	15 58 56	− 72	− 30	±32	+ 36	± 7
	GC 21595	8·4		−11° 10′·4	− 43				± 4

1371* CPD No.; Co.D. −47° 10046 puts star 10′ to north.

No.	Name and DM No.	Mag. Vis. Phot.	Type	R.A., Dec. (1900·0)	Unit 0″·001 Proper Motion R.A.	Proper Motion Dec.	P.E. Unit Weight	Parallax	P.E.
				h m s					
1375	Σ1998 s.f.	8·1	K1	15 58 56	− 68	− 28	±39	+ 22	± 8
	GC 21596	...		−11° 10′·4	− 43				± 5
1376	GC 22121	7·63	F8	16 21 18	− 292	−314	±27	+ 1	± 7
	−21° 4360	8·2		−21° 53′·6	− 282				± 4
1377	FS 16161	...	Go	17 06 19	− 69	−162	±27	+ 18	± 7
	−42° 11938	11·1		−42° 35′·8	− 69				± 5
1378	FS 16210	...	Ko	17 08 20	− 66	−130	±28	+ 21	± 6
	−51° 10781	10·7		−51° 56′·3	− 58				± 4
1379	FS 17279	...	Mo	17 39 14	+ 8	− 93	±17	+ 8	± 4
	CAZ −49°, 362	12·3		−48° 03′·0	+ 5				± 3
1380	GC 24124	8·4	Go	17 41 29	0	− 3	±30	+ 17	± 7
	Σ2211 p.	8·7		−1° 10′·6	+ 2				± 4
1381	GC 24124	8·1	Go	17 41 29	0	− 3	±33	− 4	± 8
	−1° 3389	8·7		−1° 10′·6	+ 4				± 4
1382	FS 17585	10·3	Ko	17 51 01	+ 84	−207	±38	+ 44	± 9
	−51° 11279	10·7		−51° 36′·4	+ 80				± 6
1383	FS 17884	...	G5	18 07 47	− 152	− 37	±32	+ 27	± 8
	CAZ −50°, 354	12·1		−49° 56′·9	− 142				± 6
1384	FS 18267	9·9	Ko	18 39 43	− 23	−226	±24	+ 14	± 5
	−48° 12718	11·0		−48° 13′·9	− 25				± 3
1385	FS 18300	9·9	G5	18 42 35	− 38	−185	±21	− 1	± 5
	−49° 12365	10·3		−49° 22′·2	− 23				± 3
984	UOC 53	...	...	19 06 39	+ 70	−300	±34	+ 20	± 8
	−66° 3416	11·5		−66° 55′·3	+ 62				± 4
1386	−22° 13916	...	k–m	19 18 36	− 230	−440	±29	+ 55	± 7
	LPM 703	11·9		−22° 14′·6	− 243				± 4
1387	GC 27575	9·3	G5	19 48 43	− 46	−205	±25	+ 9	± 6
	−50° 12730	9·7		−50° 12′·5	− 46				± 5
1388	ZC 18614	9·0	Go	20 00 50	− 62	−285	±30	+ 21	± 7
	−41° 13871	9·5		−41° 17′·9	− 18				± 4
1389	ZC 18733	9·2	K2	20 12 36	− 367	− 97	±18	+ 60	± 4
	−46° 13498	10·2		−46° 44′·2	− 377				± 2
1390	FS 19661	...	Mo	21 43 50	− 253	+234	±26	+ 30	± 7
	−41° 14656	11·2		−41° 43′·0	− 259				± 4
1391	ZC 19746	...	Ko	22 01 47	+ 204	− 92	±25	+ 21	± 6
	−51° 13183	10·9		−51° 10′·8	+ 172				± 4
1392	FS 19808	...	Mo	22 04 34	+ 76	− 10	±25	+ 5	± 6
	−46° 14222	12·1		−46° 28′·8	+ 62				± 3
1393	ZC 19983	9·7	F5	22 27 54	− 65	−250	±27	+ 3	± 7
	−42° 15968	10·0		−42° 33′·7	− 75				± 4

No.	Name and DM No.	Mag. Vis. Phot.	Type	R.A., Dec. (1900·0)	Unit 0·″001 Proper Motion R.A.	Proper Motion Dec.	P.E. Unit Weight	Parallax	P.E.
				h m s					
1394	FS 20007	...	K5	22 32 41	+ 237	− 41	±28	+ 1	± 7
	−47° 14277	12·2		−47° 16′·0	+ 247				± 4
1395	ZC 20324	9·4	Ko	23 04 58	+ 237	−152	±31	+ 13	± 7
	−46° 14560	10·2		−45° 59′·7	+ 234				± 5
1396	FS 20329	...	...	23 20 08	+ 469	− 44	±36	+ 41	±10
	CAZ −47°, 171	13·6		−46° 09′·5	+ 456				± 5
1397	FS 20348	...	K7	23 24 04	+ 219	− 74	±26	+ 45	± 6
	−47° 14591	11·3		−47° 35′·1	+ 216				± 4
1398	GC 32824	9·40	G5	23 31 52	+ 104	−199	±25	+ 17	± 6
	−46° 14713	10·3		−46° 01′·4	+ 107				± 4
1399	ZC 20839	8·9	Ko	23 59 16	+ 294	+ 37	±33	+ 2	± 8
	−47° 14785	11·2		−47° 37′·6	+ 275				± 4
1400	FS 20554	...	...	23 59 40	+ 183	− 20	±27	+ 6	± 7
	CAZ −51°, 214	12·1		−50° 33′·2	+ 172				± 4

Royal Observatory,
Cape of Good Hope:
1945 *June.*